Reference-Dependent Utility, Product Variety and Price Competition

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Abstract

Products such as iPhone, Coca-Cola and Tide serve as the standard against which consumers evaluate other members of the category. Empirical evidence suggests consumers care about not only the consumption utility derived from a product but also the gain-loss utility in comparison to the reference product of the category. This paper examines how reference-dependent utility affects price competition in a horizontally differentiated market where consumers’ tastes are diverse. When consumer valuations are low, the reference product is priced lower than a nonreference product. On the contrary, when consumer valuations are high, the reference product is priced higher than a nonreference product. Moreover, loss-aversion on the price dimension always intensifies competition among low valuation goods, whereas loss-aversion on the taste dimension softens competition among high valuation goods only if consumer sensitivity to the difference in match quality is above a threshold. We also find that an increase in the diversity of consumers’ tastes reduces equilibrium profits if consumer valuation is low, but has the opposite effect if consumer valuation is high. We further extend the model to explore how an increase in the popularity of the reference product affects price competition.

Keywords: Reference-Dependent Utility, Reference Price, Price Competition, Game Theory, Spokes Model.
1. INTRODUCTION

Consider a consumer planning to buy a smartphone, such as a Samsung Galaxy S6, HTC One A9 or LG V10. The consumer assesses the absolute quality and price of the product and also how the product compares with the price and quality of the iPhone 6S. Likewise, in several product categories, such as laundry detergent, shaving cream, and bath tissue, consumers’ tastes are diverse and firms offer a variety of products. Furthermore, one member of the category is viewed as the standard, and other members of the category are evaluated against this reference product when consumers make their purchase decision (e.g., Hardie, Johnson, and Fader, 1993). Thus consumer choice is affected by not only the consumption utility derived from a product but also by the gain-loss utility in comparison to the reference product (Kahneman and Tversky, 1979; Tversky and Kahneman, 1991). A meta analysis of 33 studies shows that losses loom larger than gains with the average size of the loss-aversion parameter being 1.49 (Neumann and Böckenholt 2014).

Prior marketing research has examined how a reference product or a reference price affects consumer choice in a variety of contexts and has provided estimates of the loss-aversion parameter (see Neumann and Böckenholt, 2014; Meyer and Johnson, 1995; Kalyanaram and Winer, 1995 for a review). This body of literature, however, does not investigate how the presence of a reference product could affect the competition among the plethora of products in a category. One might speculate that a reference product could potentially charge a higher price because it is the standard against which other products in a category are evaluated. Yet in some product categories, the reference product is priced lower than a nonreference product in the category. For example, Gatorade sports drink, Softsoap handwash and Gillette shaving cream are priced lower compared to other products in their category. In certain other categories the reference product is priced higher than a nonreference product. Apple, for instance, sells iPads and iPhones at higher prices than comparable tablets and smartphones. Similarly, Swatch sells its watches at higher prices than comparable watches. It is also not clear how the diversity in consumers’ tastes might moderate the effect of reference dependence on price competition. In this paper, we explore how reference-dependent consumer utility could shape price competition in a horizontally differentiated market where consumers seek
a variety of products.

To fix ideas, consider a product category where consumers’ tastes are diverse and they seek $N$ different varieties. Each of the $n$ horizontally differentiated firms in the market offer a single product. We allow for the possibility $N \geq n$, implying that all the varieties sought by consumers might not be available in the market. Consumers regard one of the $n$ products in the category as the reference product, and all the nonreference products are evaluated in relation to the price and match quality of the reference product (Tversky and Kahneman, 1991; see also Hardie, Johnson, and Fader, 1993). Because of such an evaluation process, a consumer’s decision to buy a nonreference product is affected by the intrinsic consumption utility derived from the product and also the psychological gain-loss utility in comparison to the reference product. To understand how reference-dependent utility may affect competition in a product category with numerous products, we incorporate the idea of reference-dependent utility into the spokes model (Chen and Riordan, 2007). Our theoretical analysis offers several interesting results on price competition in the presence of a reference product.

Recall that any unfavorable comparison with the price or quality of the reference product reduces the utility that consumers derive from a nonreference product. This may prompt one to think that the reference product will always be priced higher than any nonreference product in a horizontally differentiated market. On the contrary, we find that the reference product could be priced lower than other products in its category if consumer valuation of the products in the category is below a threshold. This occurs because when consumer valuation is below the threshold, all competing firms find it attractive to reduce their prices so that more consumers could gain a nonnegative surplus from buying their products. In this situation, it is profitable to sell the reference product at a lower price than any of the nonreference products. This is because when consumers compare the price of the nonreference product to that of the reference product, they experience loss aversion on the price dimension, making the nonreference product less attractive to the reference product. Hence more consumers buy the reference product and the marginal consumer is located closer to a nonreference product. Moreover, the marginal consumer does not experience a loss on match quality because she derives a higher match quality from a nonreference product than from the reference product.
This finding, however, is reversed when consumer valuation is high. Now it is profitable to sell the reference product at a higher price than a nonreference product. We observe this because when consumer valuation is above the threshold, none of the firms see a need to reduce the price to motivate consumers to buy a product. Consequently, firms do not compete aggressively on price. In this context, one can charge a higher price for the reference product and earn a higher margin. When the reference product is priced higher, its sales are lower than that of a nonreference product, and the marginal consumer is closer to the reference product. Consequently, the marginal consumers derive a higher match quality from the reference product than from their preferred nonreference products; if a marginal consumer buys a nonreference product, she would experience a psychological loss on match quality. We find that if the magnitude of loss aversion is sufficiently high, it is profitable to sell the reference product at a higher price compared to a nonreference product.

Next consider a fully covered market with $N = n$, implying that all the varieties sought by consumers are available. In this market, as consumers’ tastes become more diverse, one might expect equilibrium prices to decline because of the increase in competition. Contrary to this view, we find that an increase in the diversity in consumers’ tastes softens price competition when consumer valuation is below a threshold. To understand this finding, note that because of its lower price the reference product naturally gains more sales from the nonreference products as diversity in tastes increases. Thus, the firm selling the reference product is less motivated to aggressively cut its price and reduce its margin to sell more. Anticipating this behavior, the nonreference products also raise their absolute prices while still charging a higher price than the reference product. In a partially covered market where $N \geq n$, this finding gets reversed. We obtain this result because when diversity in consumers’ tastes increases (while keeping the number of products available in the market fixed), many consumers’ preferred products will not be available. In an attempt to motivate these consumers to buy its product, the firm supplying the reference product charges an even lower price. This intensifies price competition and the absolute prices of all the products in the market decline. We further extend the model to consider how an increase in the popularity of the reference product affects price competition. Our analysis suggests that an increase in the popularity of the reference product increases price competition when consumer valuation
is high, and the opposite result holds when consumer valuation is low.

Related Literature. Kőszegi and Rabin (2006) consider a model where a person faces some uncertainty in the future and forms a reference point based on her likely behavior in the future. She evaluates all her choices in light of the reference point. This reference point is based on the probabilistic beliefs that she holds about the relevant consumption outcome, and it is formed after she focuses on the decision determining the outcome but shortly before her consumption. They show that a consumer’s willingness to pay for a product depends on the probability with which she expected to buy the product and the price she expected to pay. An increase in the expected probability of buying a product elevates a consumer’s sense of loss if she does not buy, and this in turn increases her willingness to pay for the product. Thus, if a consumer expected a low enough price to motivate her to purchase a product, then her willingness to buy the product at a higher price increases. However, when the probability of getting the product is held constant, a decrease in expected price makes paying a higher price to be viewed a loss, and it lowers her willingness to pay a high price. As Kőszegi and Rabin (2006, p.1141) note, this model makes the extreme assumption that the reference point is fully determined by the expectations a person held in the recent past. Using a similar model of reference-dependent utility, Kőszegi and Rabin (2007) show that loss aversion causes first-order risk aversion toward all insurable risks. This occurs because the bad outcome of an uncertain lottery is viewed as a loss, whereas a fully expected premium is not considered a loss. However, a prior expectation to take risk reduces aversion to anticipated and additional risk. In contrast to these papers, there is no uncertainty in our model. Moreover, as in Tversky and Kahneman (1991), we assume that the reference product is determined à priori by factors such as order of market entry and the attributes of a nonreference product compared against those of the reference product.

Using a model where firms’ costs are stochastic, Heidhues and Kőszegi (2008) show that competing firms may charge the same price even if their actual costs are not the same. This is because if a firm charges a price that is above the expected price, consumers perceive buying the product at that price as a loss in money, and it hurts the firm’s profits. Charging a lower than expected price is not very helpful because the demand is less responsive to a price cut. Hence it is profitable for competing firms to adopt a focal-price equilibrium when
the cost asymmetry is not too high. We also consider a horizontally differentiated market, but there is no uncertainty in our model and in equilibrium firms do not charge the same price. We investigate how the rank order and absolute prices of the reference product and the nonreference products vary with consumer valuation, diversity in consumer tastes and market coverage. More recently, Zhou (2011) shows that reference-dependent utility could motivate symmetric firms to adopt a mixed pricing strategy. In Zhou’s model there is no cost uncertainty, and there are only two firms in the market. In contrast to Zhou (2011), we obtain a pure strategy solution because we consider a more general model that allows for more than two firms. Our analysis clarifies that the mixed pricing strategy equilibrium observed in Zhou (2011) is a consequence of limiting attention to two firms.

This research is also related to the work on prototypical products and prominence. He and Chen (2006) show that the featured store in a webpage can charge a higher price than any non-featured store, if consumers simultaneously search for homogeneous products. However, Armstrong, Vickers, and Zhou (2009) find that the prominent firm charges a lower price than its non-prominent rivals when consumers search sequentially in a horizontally differentiated market. Amaldoss and He (2013) show that even if prominence merely affects the probability of a product being included in a consumer’s consideration set, then it will affect the relative price of the prototypical product of a category. In contrast to these models that are based on search and consideration set formation but in keeping with the literature on reference-dependent utility, we allow consumers to experience a psychological gain-loss utility when comparing a nonreference product with a reference product. Furthermore, we show how loss aversion on price and match quality can have a differential impact on price competition depending on consumer valuation.

Thus our work contributes to the literature that examines the strategic implications of reference dependence for firm behavior. While Heidhues and Köszegi (2008) and Zhou (2011) examine when reference-dependent preferences lead to higher or lower price variations, we investigate how reference-dependent preferences could affect the rank order of the equilibrium prices of a reference product and the nonreference products. Our analysis pinpoints the reason why one does not observe a pure strategy equilibrium in Zhou (2011). We can obtain a pure strategy equilibrium in an oligopoly, but not in a duopoly. Our work also teases apart
the strategic implications of reference-dependent utility from those of product salience and product popularity.

The rest of the paper is organized as follow. Section 2 introduces the spokes model, presents a formulation of reference-dependent utility and derives the demand for the reference product and a nonreference product. Section 3 analyzes a fully covered market where all the varieties sought by consumers are available. It examines how reference-dependent utility affects the prices and profits of the reference product and a nonreference product. Section 4 considers a partially covered market where all the varieties sought by consumers are not available. This formulation helps us to assess the robustness of the results obtained in the previous section and further investigate the effect of diversity in consumers’ tastes on equilibrium behavior. Section 5 extends the models to allow for the possibility that the reference product could be more popular than any other product in the category and examines its strategic implications. Section 6 concludes the paper and outlines avenues for further research.

2. THE MODEL

Spokes Model. Consider a product category where consumers have diverse taste, and further assume that consumers seek $N$ varieties in the category. Each of the $n$ firms competing in the market offers one variety, where $2 \leq n \leq N$. Following prior literature, we model this market as a spokes network on a plane (Chen and Riordan, 2007; Amaldoss and He, 2010, 2013). Each firm indexed $j \in \{1, \ldots, n\}$ is located at the origin of the spoke for that variety, and consumers derive a base value $v$ from its product.

A unit mass of consumers is uniformly distributed on the spokes. Consumers preferring variety $j$ ($j = 1, 2, \ldots N$) are distributed on spoke $l_j$ of length $\frac{1}{2}$. Denote the consumer on spoke $j$ at distance $x$ from the origin by $(l_j, x)$ where $x \in [0, \frac{1}{2}]$. If this consumer buys the local product $j$, she will derive the (indirect) utility $v - tx - p_j$, where $t$ is her sensitivity to product characteristics and $p_j$ is the price of the product. Instead, if the consumer chooses to buy another product $k$, she has to travel a distance of $(1 - x)$ because the consumer is $\frac{1}{2} - x$ units away from the center of the spokes network and the nonlocal product is an additional $\frac{1}{2}$ unit away, that is $\frac{1}{2} - x + \frac{1}{2} = 1 - x$. Hence the (indirect) utility derived from the nonlocal product $k$ will be $v - t(1 - x) - p_k$. The marginal consumer who is indifferent between the
two products is at distance $\frac{1}{2} + \frac{p_k - p_j}{2t}$ from product $j$.

When making a purchase decision, consumers typically choose from a small consideration set (Nedungadi, 1990; Hauser and Wernerfelt, 1990). Consistent with this observation, the literature on the spokes model assumes that consumers consider at most two products (Chen and Riordan, 2007; Amaldoss and He, 2010, 2013). In our model, the first-preferred product of a consumer is the local variety corresponding to the spoke in which she resides. The second-preferred product is one of the nonlocal varieties available in the market, and it is exogenously fixed à priori. Letting the consideration set size be two helps us to obtain a pure-strategy equilibrium. Next we add the idea of reference-dependent utility to the spokes model.

**Reference-Dependent Utility.** Reference dependence influences the utility consumers derive from a nonreference product in two ways. When a consumer finds that a nonreference product’s price is higher than the reference product’s price, she experiences a psychological disutility, which increases with the size of the price difference. The consumer also incurs
a psychological disutility if the intrinsic utility of a nonreference product is lower than the match utility of the reference product. To fix ideas, let \( z \) be the reference product of the category and \( k \) be one of the nonreference products. If the consumer located at \((l_z, x)\) buys the reference product at price \( p_z \), she will derive the following (indirect) utility:

\[
U(l_z, x, p_z) = v - tx - p_z. \tag{1}
\]

However, if the consumer purchases the nonreference product \( j \) at price \( p_j \), the (indirect) utility derived from the product is given by:

\[
v - p_j - t(1 - x) - \lambda \max\{0, p_j - p_z\} - \mu t \max\{0, 1 - 2x\}, \tag{2}
\]

where \( \lambda > 0 \) is a measure of the consumer’s sensitivity to price difference with the reference product, and \( \mu > 0 \) is a measure of the consumer’s sensitivity to match utility difference with the reference product. In other words, \( \lambda \) and \( \mu \) are loss aversion parameters pertaining to price and match quality. In the above expression, \( v - p_j - t(1 - x) \) gives the intrinsic surplus the consumer enjoys from buying the nonreference product instead of the reference product. If the nonreference product is higher priced, the consumer suffers a disutility, which is given by \( \lambda \max\{0, p_j - p_z\} \). In the event the nonreference product offers a lower match utility, the consumers sustains a disutility given by \( \mu t \max\{0, 1 - 2x\} \). In this framework, reference dependence does not add to the utility of the reference product, but reduces the utility of the nonreference product when it compares unfavorably with the reference product on price or match utility. Figure 1 illustrates a market where consumers seek \( N = 10 \) varieties but only \( n = 3 \) products are available. Of the available products, \( z \) is the reference product while \( k \) and \( j \) are the nonreference products.

**Consumer Demand.** Using this formulation of reference-dependent utility, we derive the demand for the reference product and the demand for a nonreference product. Demand for the reference product. Three segments of consumers contribute to the demand. The first segment is comprised of consumers whose first-preferred product and second-preferred product are both available, and reference product \( z \) is the first-preferred product. The second segment includes consumers whose first-preferred product is the reference product \( z \) but their second-preferred product is not available. The third segment is made up
of consumers whose first-preferred product is not available and the reference product $z$ is their second-preferred product. Figure 2 illustrates the different segments of consumers from which the reference product draws its demand.

**Segment 1 (R).** Depending on the relative prices, consumers in this segment decide on whether to buy the reference product $z$ or the nonreference product $k$ in their consideration set. The location of the marginal consumer who is indifferent between the two products is given by:

$$x = \begin{cases} 
\min \left\{ \frac{1}{2}, \frac{1}{2} - \frac{1+\lambda}{2t} (p_z - p_j) \right\}, & \text{for } p_z \leq p_j; \\
\max \left\{ 0, \frac{1}{2} - \frac{1}{2t(1+\mu)} (p_z - p_j) \right\}, & \text{for } p_z > p_j.
\end{cases} \tag{3}$$

Note that for any consumer located on $l_z$, product $j$ is her second-preferred product with probability $\frac{1}{N-1}$, where $j \in \{1, \ldots, n\}$ and $j \neq z$. Because consumers are evenly distributed on all spokes, the density of such consumers is $\frac{2}{N}$.\(^1\) Hence, the demand from these consumers for product $z$ is:

$$\frac{2}{N} \frac{1}{N-1} \sum_{k \neq z, k \in \{1, \ldots, n\}} x. \tag{4}$$

\(^1\)Later, we consider the situation where more consumers reside on the spoke with the reference product, implying that the reference product is the first-preferred product for a larger mass of consumers.
Segment 2 (R). Because the second-preferred product of the consumers in this segment is not available, they need to decide whether to buy the reference product or nothing. For any consumer on spoke \( l_z \), her second-preferred variety is not available with conditional probability \( \frac{N-n}{N-1} \). Hence, the demand for product \( z \) from these consumers is given by:

\[
\frac{2}{N} \frac{N-n}{N-1} \min \left\{ \max \left\{ 0, \frac{v - p_z}{t} \right\}, \frac{1}{2} \right\}.
\]

\( \text{(5)} \)

Segment 3 (R). Consumers in this segment consider reference product \( z \) to be their second-preferred product. We know that for a consumer on spoke \( l_h \), \( h \notin \{1, \ldots, n\} \), product \( z \) is the second-preferred product with probability \( \frac{1}{N-1} \). Furthermore, the density of such consumers is \( \frac{2}{N} (N-n) \) and the demand for product \( z \) from these consumers is:

\[
\frac{2}{N} \frac{N-n}{N-1} \min \left\{ \max \left\{ 0, \frac{v - p_z}{t} - \frac{1}{2} \right\}, \frac{1}{2} \right\}.
\]

\( \text{(6)} \)

Across the three segments, the total demand for the reference product \( z \) is

\[
d_z = \frac{2}{N} \frac{1}{N-1} \left\{ (n-1) x + (N-n) \min \left\{ \max \left\{ 0, \frac{v-p_z}{t} \right\}, \frac{1}{2} \right\} \right\} \\
+ (N-n) \min \left\{ \max \left\{ 0, \frac{v-p_z}{t} - \frac{1}{2} \right\}, \frac{1}{2} \right\}
\]

\( \text{(7)} \)

Demand for a nonreference product. The demand for a given nonreference product \( j \) emanates from four segments of consumers. The first segment consists of consumers whose first-preferred and second-preferred varieties are available, and one of them is the reference product. The second segment is comprised of consumers whose first-preferred and second-preferred products are nonreference products. For the third segment of consumers, the nonreference product is the first-preferred product and their second-preferred variety is not available. Finally, for the fourth segment of consumers, a nonreference product is the second-preferred variety and the first-preferred variety is not available. Figure 3 presents the four segments of consumers discussed above.

Segment 1 (NR). For a consumer located on \( l_z \), nonreference product \( j \) is her second-preferred product with probability \( \frac{1}{N-1} \); likewise, for any consumer located on \( l_j \), reference product \( z \) is her second-preferred product with probability \( \frac{1}{N-1} \). The density of such consumers is \( \frac{2}{N} \). Hence, the demand from these consumers for product \( j \) is:

\[
\frac{2}{N} \frac{1}{N-1} (1-x).
\]

\( \text{(8)} \)
**Fig. 3. Segmentation of a Nonreference Product’s Consumers**

Segment 2 (NR). Next for a consumer located on $l_j$, nonreference product $k$ is her second-preferred product with probability $\frac{1}{N-1}$, where $k \in \{1, \ldots, n\}$ and $k \neq z, j$. Furthermore, because the nonreference product $k$ could be any product other than products $z$ and $j$, the demand from these consumers for product $j$ is:

$$2 \left(\frac{n-2}{N(N-1)}\right) \max \left\{ \min \left(\frac{1}{2} + \frac{p_k - p_j}{2t}, 1\right), 0 \right\}. \quad (9)$$

Segment 3 (NR). Recall that the second-preferred variety is not available for a consumer in this segment. For a consumer on spoke $l_j$, her second-preferred variety is not available with conditional probability $\frac{N-n}{N-1}$; so the demand for nonreference product $j$ from these consumers is given by:

$$2 \frac{N-n}{N(N-1)} \min \left\{ \max \left(0, \frac{v - p_j}{t}\right), \frac{1}{2} \right\}. \quad (10)$$

Segment 4 (NR). For consumers in this segment their first-preferred product is not available. Note that for a consumer on spoke $l_h$, $h \notin \{1, \ldots, n\}$, product $j$ is the second-preferred product with probability $\frac{1}{N-1}$, and that the density of such consumers is $\frac{2}{N}(N-n)$. Hence the demand for nonreference product $j$ from these consumers is:

$$2 \frac{N-n}{N(N-1)} \min \left\{ \max \left(0, \frac{v - p_j}{t} - \frac{1}{2}\right), \frac{1}{2} \right\}. \quad (11)$$
Upon aggregating the demand for the four segments, we have:

\[
d_j = \frac{2}{N} \frac{1}{N-1} \left\{ (1-x) + (n-2) \max \left\{ \min \left( \frac{1}{2} + \frac{p_{i-k} - p_j}{2t}, 1 \right), 0 \right\} 
+ (N-n) \min \left\{ \max \left( 0, \frac{v-p_j}{t} \right), \frac{1}{2} \right\} + (N-n) \min \left\{ \max \left( 0, \frac{v-p_j}{t} - \frac{1}{2} \right), \frac{1}{2} \right\} \right\}
\]

(12)

When consumers have reference-dependent utility as illustrated in equation (2), the location of the marginal consumer \((x)\) depends on the rank order of the prices of the reference product and the nonreference products. Provided that \(\frac{|p_r - p_{i-k}|}{t} \leq 1\), the demand for the reference product and that for a nonreference product can be written as follows:

**Case a:** \(p_z < p_j\)

\[
d_z = \begin{cases} 
\frac{2}{N} \frac{1}{N-1} \left( (n-1) \left( \frac{1}{2} - \frac{1+\lambda}{2t} (p_z - p_j) \right) + (N-n) \frac{v-p_z}{t} \right) & \text{for } \frac{1}{2} < \frac{v-p_z}{t} < 1 \\
\frac{2}{N} \frac{1}{N-1} \left( (n-1) \left( \frac{1}{2} + \frac{1+\lambda}{2t} (p_z - p_j) \right) + (N-n) \right) & \text{for } \frac{v-p_z}{t} \geq 1 
\end{cases}
\]

(13)

\[
d_j = \begin{cases} 
\frac{2}{N} \frac{1}{N-1} \left( \frac{1}{2} + \frac{1+\lambda}{2t} (p_z - p_j) + (n-2) \left( \frac{1}{2} + \frac{p_k - p_j}{2t} \right) + (N-n) \frac{v-p_j}{t} \right) & \text{for } \frac{1}{2} < \frac{v-p_j}{t} < 1 \\
\frac{2}{N} \frac{1}{N-1} \left( \frac{1}{2} + \frac{1+\lambda}{2t} (p_z - p_j) + (n-2) \left( \frac{1}{2} + \frac{p_k - p_j}{2t} \right) + (N-n) \right) & \text{for } \frac{v-p_j}{t} \geq 1 
\end{cases}
\]

(14)

**Case b:** \(p_z > p_j\)

\[
d_z = \begin{cases} 
\frac{2}{N} \frac{1}{N-1} \left( (n-1) \left( \frac{1}{2} - \frac{p_z - p_j}{2t(1+\mu)} \right) + (N-n) \frac{v-p_z}{t} \right) & \text{for } \frac{1}{2} < \frac{v-p_z}{t} < 1 \\
\frac{2}{N} \frac{1}{N-1} \left( (n-1) \left( \frac{1}{2} - \frac{p_z - p_j}{2t(1+\mu)} \right) + (N-n) \right) & \text{for } \frac{v-p_z}{t} \geq 1 
\end{cases}
\]

(15)

\[
d_j = \begin{cases} 
\frac{2}{N} \frac{1}{N-1} \left( \frac{1}{2} + \frac{1}{2t(1+\mu)} (p_z - p_j) + (n-2) \left( \frac{1}{2} + \frac{p_k - p_j}{2t} \right) + (N-n) \frac{v-p_j}{t} \right) & \text{for } \frac{1}{2} < \frac{v-p_j}{t} < 1 \\
\frac{2}{N} \frac{1}{N-1} \left( \frac{1}{2} + \frac{1}{2t(1+\mu)} (p_z - p_j) + (n-2) \left( \frac{1}{2} + \frac{p_k - p_j}{2t} \right) + (N-n) \right) & \text{for } \frac{v-p_j}{t} \geq 1 
\end{cases}
\]

(16)

In our model, both firms simultaneously set prices to maximize their respective profits. After observing the prices, consumers make their purchase decisions. We assume that the marginal cost of producing a product is the same for all products, and set it equal to zero without loss of generality. We examine the pure-strategy equilibrium of the game to understand the strategic behavior of competing firms.
3. ANALYSIS OF A FULLY COVERED MARKET

In this section, we analyze a fully covered market to understand how reference-dependent utility can influence the behavior of competing firms. Later, we investigate a partially covered market to explore how market coverage might moderate the effect of reference dependence in a competitive market. We also highlight how an increase in the popularity of the reference product can shape equilibrium behavior.

Prices

We begin our analysis by examining the equilibrium prices in a fully covered market with at least three products \((N = n \geq 3)\). In the absence of reference-dependent utility, one may expect all competing firms to charge the same price. Consistent with this intuition, in equilibrium all firms charge the price \(t\) when \(\lambda = 0\) and \(\mu = 0\). This observation may raise some questions. For example, how does reference dependence affect the rank order of the prices of the reference product and a nonreference product? How does reference dependence influence the absolute prices of the reference product and the nonreference product? We examine these issues below.

**Relative Prices.** Because reference dependence hurts the utility derived from a nonreference product, one could argue that the reference product might be higher priced than a nonreference product. Counter to this line of thinking, we have the following finding:

**Proposition 1** The reference product is lower priced than a nonreference product when \(N = n \geq 3\) and \(v_0 < v < \min\{v_1, v_2\}\), where:

\[
\begin{align*}
v_0 &\equiv \frac{3t}{2} \frac{2n + \lambda - 1}{2n + 3\lambda - 1}, \\
v_1 &\equiv \frac{t}{2} \frac{3(2n - 1) + 2\mu(4n - 5) + 2\mu^2(n - 2)}{2n - 4\mu + 2n\mu - 1}, \\
v_2 &\equiv \frac{t}{2} \frac{4n - 5\mu + 4n\mu - 2}{2n - 4\mu + 2n\mu - 1}.
\end{align*}
\]

To follow the intuition for this finding, note that when \(v_0 < v < \min\{v_1, v_2\}\), consumer valuation is low and it is not worthwhile for a consumer to travel all the way from one end of a spoke to the end of another spoke to buy a nonlocal product. Recognizing the low valuation
of products, firms are motivated to reduce the price so that consumers could gain a surplus on purchasing their products. The firm offering the reference product finds it profitable to charge an even lower price for the reference product compared to the price of a nonreference product. To see why, note that a lower priced reference product makes a nonreference product less attractive, because consumers would experience loss aversion on the price dimension. Consequently, more consumers purchase the reference product and the marginal consumer, who is indifferent between the reference product and a nonreference product lies, close to the nonreference product. Moreover, since the marginal consumer derives a higher level of functional utility from the nonreference product, she does not experience any disutility on the taste dimension when she compares the reference product with the nonreference product. Thus the size of $\mu$ does not influence her utility from the reference product.

Though all consumers derive a positive surplus on purchasing their local product, some consumers may not obtain a positive surplus from buying a nonlocal product if the base valuation is not high enough. Hence, if consumer valuation is below $v_0$, each firm would become a local monopolist and charge $v - \frac{t}{2}$. But if the valuation is not low enough, the reference product can charge a slightly lower price and gain sales from the nonreference products. We note that when consumers become more sensitive to the difference in price, the threshold $v_0$ decreases with $\lambda$. Specifically, we have:

$$\frac{\partial v_0}{\partial \lambda} = -3t \frac{2n - 1}{(2n + 3\lambda - 1)^2} < 0.$$  (17)

Thus, the valuation region in which the reference product is lower priced expands when consumer sensitivity to price difference increases. Next, we explore when it would be profitable for the reference product to be higher priced.

**Proposition 2** The reference product is higher priced than a nonreference product when $N = n \geq 3$ and $v > \min\{v_1, v_2\}$.

This finding is the exact opposite of the result reported in Proposition 1. When consumer valuation is high, consumers could potentially buy any of the products in their consideration set and gain a surplus. Hence firms are not motivated to aggressively cut their prices. If the reference product is higher priced, the marginal consumer does not experience psychological
loss on the price dimension and thus $\lambda$ does not affect the utility she derives from the reference product. Furthermore, because fewer consumers buy the high-priced reference product, the marginal consumer is located on the spoke in which the reference product lies, and she derives a higher functional utility from the reference product compared to the nonreference product. This suggests that the marginal consumer would face disutility on the taste dimension if she does not purchase the reference product, and it encourages the firm supplying the reference product to charge a higher price. Thus $\mu$ comes to influence equilibrium prices in the high valuation region, and the reference product is priced higher.

The region in which the reference product is higher priced shrinks with consumer’s sensitivity to the difference in match qualities of the two products. To see why, note that the marginal consumer located on $l_z$ would not purchase the nonlocal product unless her valuation is high enough. Furthermore, an increase in $\mu$ reduces the utility derived from a nonreference product given its lower match quality. To offset this disutility, the threshold value $\min\{v_1, v_2\}$ increases with $\mu$. Specifically, we have:

$$
\frac{\partial v_1}{\partial \mu} = 3t \frac{2n - 1}{(4\mu - 2n - 2n\mu + 1)^2} > 0,
$$

$$
\frac{\partial v_2}{\partial \mu} = t \frac{n + 2n^2 - 1 + 2\mu (n - 2) (2n - 2\mu + n\mu - 1)}{(2n - 4\mu + 2n\mu - 1)^2} > 0.
$$

(18)

**Absolute Prices.** The preceding analysis shows how the rank order of prices is moderated by consumer valuation. Recall that in the absence of reference dependence all firms will charge the price $t$ for their products. On examining how the absolute prices change with the magnitude of consumers’ susceptibility to reference dependence, we have the following result:

**Proposition 3** The equilibrium prices of all products decrease with the magnitude of loss aversion on price ($\lambda$) and increase with $N$ when $N = n \geq 3$ and $v_0 < v < \min\{v_1, v_2\}$. On the contrary, the equilibrium prices of all products increase with loss aversion on match quality ($\mu$) and decrease with $N$ when $N = n \geq 3$ and $v > \min\{v_1, v_2\}$.

From Proposition 1, we know that when valuation is low the firm offering the reference product finds it profitable to undercut the price of a nonreference product because consumers evince loss aversion on the price dimension. As $\lambda$ grows in size, the price of the reference
product falls further below $t$. In response, a firm offering a nonreference product also decreases its price below $t$ though still charging a higher price than the reference product. On the other hand, when valuation is high, the marginal consumer purchasing the nonreference product experiences loss on the taste dimension, but not on the price dimension. Consequently, equilibrium prices are affected by $\mu$, not $\lambda$. Unlike loss aversion on price, loss aversion on match quality softens price competition and helps all firms to raise their prices.

Notice that in a fully covered market when consumers’ tastes become more diverse, the number of competitors in the market increases. This could lead one to conjecture that prices might decline with an increase in $N$. Yet when consumer valuation is low we observe the opposite effect. This occurs because the firm selling the reference product needs to make a trade-off between lower margin and higher sales. When $N$ is large, the firm has an opportunity to slightly improve its margin on the reference product without compromising too much on the total sales gained from all of its competitors. Consistent with our intuition, when consumer valuation is high, an increase in $N$ reduces the equilibrium prices. Recall that when consumer valuation is high, the firm selling the reference product charges a higher price than a nonreference product. In this context an increase in $N$ motivates the firm to reduce its price so that it can gain more sales from its competitors and potentially raise its total profits.

**Discussion.** The above analysis highlights how reference dependence changes the rank order of prices and the absolute prices. It is important to note that the mechanism by which reference dependence induces these changes is very different from how product salience affects equilibrium prices. Salience raises the probability of a product being included in the consideration set of consumers and reduces the proportion of consumers not aware of their first or second-preferred products. This affects the price sensitivity of the various segments of consumers that make up the market and in turn the equilibrium prices (Amaldoss and He, 2013). Reference dependence, on the other hand, directly affects the utility that consumers derive from the nonreference product by making them loss averse to price or match quality.

Next, in developing our model we considered a fully covered market with at least three products ($N = n \geq 3$). This helped us to obtain a pure strategy solution and clarify the intuition for the results. The standard Hotelling model is a special case of the spokes model
with \( N = n = 2 \). Notice that when \( n = 2 \), however, there is no pure strategy solution though the game has a mixed strategy solution (Zhou, 2011).\(^2\) Our analysis brings to the fore an important issue related to the existence of a pure strategy equilibrium in the presence of consumers with reference-dependent utility: the nonexistence of a pure strategy solution is limited to the standard Hotelling model, and we obtain a pure strategy solution when \( n \geq 3 \). On further probing, we find that when \( n = 2 \), the nonreference product fully internalizes the disutility induced by reference dependence at any given price, forcing both the competing firms to adopt a mixed pricing strategy rather than a pure strategy. On the other hand, when \( n \geq 3 \), it is not possible for all the nonreference products to cooperatively internalize the disutility induced by reference dependence and hence we obtain a pure strategy solution.

**Profits**

The preceding analysis shows how reference dependence affects equilibrium prices and potential sales. Next we first discuss how reference dependence affects the rank-order of the profits derived from the reference product and a nonreference product, and then how it changes the absolute profits from these two products.

**Relative Profits.** On comparing the equilibrium profits of the reference product and of a nonreference product, we have the following finding:

**Proposition 4** The reference product makes strictly higher profits than a nonreference product if \( v_0 < v < \min\{v_1, v_2\} \). However, when \( v > \min\{v_1, v_2\} \), the reference product earns higher profits than a nonreference product only if \( \mu > \frac{2n-1}{(n-1)(n-2)} \).

In the low valuation region, the firm supplying the reference product charges a lower price and gains market share from each of the nonreference products. As the gain in sales more than compensates for the reduced margin, the reference product is better off than the nonreference products in the low valuation region. In the high valuation region, however, the reference product earns more profits than a nonreference product only when \( \mu \) is sufficiently strong. To understand why, note that in the high valuation region, the reference product charges a higher price and loses some market share to each of the nonreference products. If

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\(^2\)Please see Appendix B for details.
\( \mu \) is higher than the threshold specified in the above proposition, the reference product earns more profits than any of the nonreference products because the increase in price more than offsets the loss in sales. Casual observation suggests that in many product categories more consumers prefer the reference product compared to a nonreference product. As we show later, when we allow for such a possibility, the threshold value of \( \mu \) drops much below the value in Proposition 4.

**Absolute Profits** Having studied the relative profits, now we investigate how the absolute profits change with reference dependence and to what extent this effect varies with consumer valuation. We have the following result.

**Proposition 5**  

1. When \( v_0 < v < \min\{v_1, v_2\} \), the profits of the nonreference products decrease with \( \lambda \), whereas the profits of the reference product do not always decline with \( \lambda \) if there is sufficient variety of products with \( N \geq 13 \).  
2. When \( v > \min\{v_1, v_2\} \), the equilibrium profits of both the reference product and the nonreference product increase with \( \mu \).

Intuitively, when consumer valuation is low, an increase in \( \lambda \) makes consumers more sensitive to a price difference, and this hurts the profits of a nonreference product. Recall that when consumer valuation is low, the reference product charges a lower price than a nonreference product, and an increase in \( \lambda \) can have a negative effect on its profits as well. However, when the diversity in consumers' tastes is sufficiently large, the reference product can wean sales from larger number of nonreference products with a small cut in its price. The resulting larger sales for a small reduction in margin can increase reference product’s profits. On the contrary, when consumer valuation is high, an increase in \( \mu \) increases consumers' sensitivity to the difference in match quality and increases the profits of both the reference and the nonreference products by softening competition. Notice that though an increase in \( N \) is a blessing for the reference product in the low valuation region, it turns out to be a curse in the high valuation region. This is because in the low valuation region it weakens the price competition induced by \( \lambda \), whereas in the high-valuation region it weakens the competition mitigating effect of \( \mu \). Therefore, profits decline with \( N \) in the high-valuation region. Having examined the implications of reference-dependent utility for a fully covered market, we now
shift attention to its ramifications for a partially covered market.

4. ANALYSIS OF A PARTIALLY COVERED MARKET

In some product categories all the varieties that consumers seek may not be available, and hence some consumers may not have an opportunity to buy their most preferred product varieties. In this section, we seek to understand the strategic implications of such a partially covered market. According to Proposition 3, in a fully covered market when consumers’ tastes become more diverse, equilibrium price decreases if consumer valuation is low but increases if consumer valuation is high. An important implication of this finding is that as $N$ increases, equilibrium profits increase when valuation is low, but decline when consumer valuation is high. On investigating the effect of $N$ on equilibrium price in a partially covered market, we have the following result.

**Proposition 6** In a partially covered market $(N > n \geq 3)$, equilibrium profits decline with diversity in consumer taste $(N)$ if consumer valuation is low, but increase with diversity in taste if consumer valuation is high.

To understand the intuition for this finding note that the demand for the reference product comes from three segments of consumers: a competitive segment and two monopoly segments. Segment 1 (R) is the competitive segment where consumers could choose between their first and second-preferred products (see equation 4). On the other hand, Segment 2 (R) and Segment 3 (R) are the monopoly segments where either consumers’ first or second-preferred product is not available (see equations 5 and 6, respectively). Consequently, consumers in these two monopoly segments will purchase the reference product as long as it yields a nonnegative surplus. Likewise the demand for a nonreference product arises from two competitive and two monopoly segments. Consumers in Segment 1 (NR) and Segment 2 (NR) choose among the products in their consideration set, whereas consumers in segments Segment 3 (NR) and Segment 4 (NR) do not have this privilege because one of the products in their consideration set is not available. Hence, in a partially covered market, each competing firm needs to carefully balance the gains from the monopoly segments against those from the competitive markets while choosing its strategy.
When consumer valuation is low, the demand from monopoly segments is more elastic than the demand from competitive segments. This is because when a firm reduces its price, it sells to more consumers in a monopoly segment. By contrast, the location of the marginal consumer in a competitive segment does not move or moves less due to competitive reaction of other firms. Therefore the firm is less motivated to cut price to increase sales from the competitive segment. Now if we keep $n$ constant and increase $N$, the relative size of the monopoly segments grows while the size of the competitive segments shrinks. Consequently, the overall demand is more elastic to price change provided $p < v - t$. Hence profits decline with $N$ when consumer valuation is low. On the other hand, when consumer valuation is high all consumers could gain some surplus by buying any product in their consideration set. Consequently, consumers in a monopoly segment are completely price inelastic. Since the relative size of the monopoly segments increase with $N$, firm’s profits also increase with $N$.

Proposition 6 could make one wonder whether the effect of reference dependence in a partially covered market is qualitatively different from that in a fully covered market. The answer is no. In a partially covered market, as in a fully covered market, the reference product is lower priced than a nonreference product when consumer valuation is low (compare with Proposition 1). Furthermore, the reference product is higher priced than a nonreference product when consumer valuation is high (compare with Proposition 2). We prove these claims in Appendix B.

5. MODEL OF A POPULAR REFERENCE PRODUCT

The preceding analysis conservatively assumes that the proportion of consumers who prefer the reference product is the same as that for a nonreference product. This symmetry assumption helped us to tease out the effect of reference dependence on firms’ competitive strategies. In reality, the reference product is often the most popular product in its category, implying that more consumers prefer the reference product to a nonreference product. Now we relax the symmetry assumption to examine how an increase in the popularity of the reference product may affect equilibrium behavior.

Consider a spokes model with $N > n \geq 3$. Further assume that $\alpha$ proportion of consumers
reside on spoke $z$ where the reference product is located, and that $\frac{1}{N} \leq \alpha < 1$. Note that the partially covered market model discussed in the previous section is a special case of this general model with $\alpha = \frac{1}{N}$. The remaining $1 - \alpha$ consumers are evenly distributed on the other $N - 1$ spokes. Next we derive the demand for the popular reference product and that for a nonreference product.

**Demand for the Reference Product.** The reference product $z$ draws its demand from three segments of consumers. First, a segment of consumers whose first-preferred product and the second-preferred product are both available (Segment 1a (R)). Second, a segment of consumers whose first-preferred product is $z$ but the second-preferred product is not available (Segment 2a (R)). Third, a segment of consumers whose first-preferred product is not available and $z$ is the second-preferred product (Segment 3a (R)). Below we discuss each of these segments.

**Segment 1a (R).** Consumers in this segment regard the reference product as the first-preferred product, and both the products in their consideration set are available. For a consumer located on $l_z$, product $j$ is her second-preferred product with probability $\frac{1}{N-1}$, where $j \in \{1, \ldots, n\}$ and $j \neq z$. Thus, the conditional probability that a consumer located on $l_z$ has both her first-preferred and second-preferred products available is $\frac{n-1}{N-1}$. The demand depends on whether the marginal consumer is located on spoke $z$ or spoke $j$. If the marginal consumer $x$ is located on spoke $z$, the demand comes from only consumers on spoke $z$ and the corresponding demand is $\frac{2\alpha x (n-1)}{N-1}$. But if the marginal consumer is on spoke $j$, demand comes from two sources. First, consumers located on spoke $z$ would buy product $z$, and this demand is given by $\frac{1}{2} \alpha \frac{n-1}{N-1} = \frac{\alpha (n-1)}{N-1}$. Second, some of the consumers located on spoke $j$ could also buy product $z$, and this demand is $\frac{2(1-\alpha) n-1}{N-1} (x - \frac{1}{2}) = \frac{2(1-\alpha)(n-1)}{(N-1)^2} (x - \frac{1}{2})$. Taken together, the demand from this segment for reference product $z$ is as follows:

\[
\frac{\alpha (n-1)}{N-1} + \frac{2(1-\alpha)(n-1)}{(N-1)^2} (x - \frac{1}{2}), \quad \text{for } x > \frac{1}{2};
\]
\[
\frac{2\alpha x (n-1)}{N-1}, \quad \text{for } x \leq \frac{1}{2}.
\]

where the location of the marginal consumer $x$ is as given in equation (3).

**Segment 2a (R).** For a consumer in this segment, the reference product is the second-preferred variety, and her first-preferred variety is not available. Note that for a consumer on spoke $l_z$ her second-preferred variety is not available with conditional probability $\frac{N-n}{N-1}$.
Hence, the demand for reference product $z$ from these consumers is given by:

$$2\alpha \frac{N-n}{N-1} \min \left\{ \max \left(0, \frac{v-p_z}{t} \right), \frac{1}{2} \right\}.$$  \hspace{1cm} (20)

**Segment 3a (R).** Consumers in this segment view the reference product as the second-preferred product and their first-preferred variety is not available. For a consumer on spoke $l_h$, $h \notin \{1, \ldots, n\}$, product $z$ is the second-preferred product with probability $\frac{1}{N-1}$. Furthermore, the density of such consumers is $2(1-\alpha) (N-n)$ and the demand for reference product $z$ from these consumers is:

$$2 \left(1-\alpha\right) \frac{N-n}{N-1} \min \left\{ \max \left(0, \frac{v-p_z}{t} - \frac{1}{2} \right), \frac{1}{2} \right\}.$$  \hspace{1cm} (21)

Upon aggregating, the total demand for reference product $z$ is as follows:

$$d_z = \begin{cases} \frac{\alpha(n-1) + 2(1-\alpha)(n-1)}{N-1} \left( x - \frac{1}{2} \right) + 2\alpha \frac{N-n}{N-1} \min \left\{ \max \left(0, \frac{v-p_z}{t} \right), \frac{1}{2} \right\} & \text{for } x > \frac{1}{2}, \\ \frac{2\alpha(n-1)}{N-1} + \frac{2(1-\alpha) N-n}{N-1} \min \left\{ \max \left(0, \frac{v-p_z}{t} - \frac{1}{2} \right), \frac{1}{2} \right\} & \text{for } x \leq \frac{1}{2}, \\ \frac{2\alpha x(n-1)}{N-1} + 2\alpha \frac{N-n}{N-1} \min \left\{ \max \left(0, \frac{v-p_z}{t} \right), \frac{1}{2} \right\} & \text{for } x > \frac{1}{2}, \\ + \frac{2\alpha x(n-1)}{N-1} + \frac{2(1-\alpha) N-n}{N-1} \min \left\{ \max \left(0, \frac{v-p_z}{t} - \frac{1}{2} \right), \frac{1}{2} \right\} & \text{for } x \leq \frac{1}{2}. \end{cases}$$  \hspace{1cm} (22)

Demand for a nonreference product. The demand for a nonreference product $j$ comes from four segments of consumers. First, a segment of consumers whose first-preferred product and the second-preferred product are both available and one of them is the reference product $z$ (Segment 1a (NR)). Second, a group of consumers whose first-preferred product and the second-preferred product are both nonreference products (Segment 2a (NR)). Third, a segment of consumers whose first-preferred product is $j$ but the second-preferred product is not available (Segment 3a (NR)). Fourth, a segment of consumers whose first-preferred product is not available and $j$ is the second-preferred product (Segment 4a (NR)).

**Segment 1a (NR).** For a consumer located on $l_z$, product $j$ is her second-preferred product with probability $\frac{1}{N-1}$. Similarly, for any consumer located on $l_j$, product $z$ is her second-preferred product with probability $\frac{1}{N-1}$. The density of such consumers is $\frac{2\alpha}{N}$ if the marginal consumer is on spoke $l_z$, but it is $\frac{2(1-\alpha)}{N}$ if the marginal consumer is on spoke $l_j$. Hence, the demand from these consumers for nonreference product $j$ is:

$$d_j = \begin{cases} \frac{2(1-\alpha)}{(N-1)^2} \left( 1 - x \right), & \text{for } x > \frac{1}{2}; \\ \frac{2\alpha}{N-1} \left( \frac{1}{2} - x \right) + \frac{1-\alpha}{(N-1)^2}, & \text{for } x \leq \frac{1}{2}. \end{cases}$$  \hspace{1cm} (23)
Segment 2a (NR). Next for any consumer located on \( l_j \), product \( k \) is her second-preferred product with probability \( \frac{1}{N-1} \), where \( k \in \{1, \ldots, n\} \) and \( k \neq z, j \). Hence, the demand from these consumers for nonreference product \( j \) is:

\[
\frac{2(1-\alpha)}{N-1} \frac{n-2}{N-1} \max \left\{ \min \left( \frac{1}{2} + \frac{p_k - p_j}{2t}, 1 \right), 0 \right\}. \tag{24}
\]

Segment 3a (NR). Consumers in this segment consider product \( z \) as their second-preferred product. We know that for a consumer on spoke \( l_h \), \( h \notin \{1, \ldots, n\} \), product \( z \) is the second-preferred product with probability \( \frac{1}{N-1} \). Furthermore, the density of such consumers is \( \frac{2(1-\alpha)}{N}(N-n) \) and the demand for nonreference product \( j \) from these consumers is:

\[
\frac{2(1-\alpha)}{N-1} \frac{N-n}{N-1} \min \left\{ \max \left( 0, \frac{v - p_j}{t} \right) - \frac{1}{2} \right\}. \tag{25}
\]

Segment 4a (NR). For a consumer on spoke \( l_h \), \( h \notin \{1, \ldots, n\} \), product \( j \) is the second-preferred product with probability \( \frac{1}{N-1} \). Furthermore, the density of such consumers is \( \frac{2(1-\alpha)}{N}(N-n) \) and the demand for product \( j \) from these consumers is:

\[
\frac{2(1-\alpha)}{N-1} \frac{N-n}{N-1} \min \left\{ \max \left( 0, \frac{v - p_j}{t} \right) - \frac{1}{2} \right\}. \tag{26}
\]

Across the four segments, the total demand for the nonreference product \( j \) is

\[
d_j = \begin{cases} 
\frac{2(1-\alpha)}{(N-1)^2} (1 - x) + \frac{2(1-\alpha)}{N-1} \frac{n-2}{N-1} \max \left\{ \min \left( \frac{1}{2} + \frac{p_k - p_j}{2t}, 1 \right), 0 \right\}, & \text{for } x > \frac{1}{2}; \\
\frac{2(1-\alpha)}{N-1} \left( \frac{1}{2} - x \right) + \frac{1-\alpha}{(N-1)^2} + \frac{2(1-\alpha)}{N-1} \frac{n-2}{N-1} \max \left\{ \min \left( \frac{1}{2} + \frac{p_k - p_j}{2t}, 1 \right), 0 \right\} + \frac{2(1-\alpha)}{N-1} \frac{N-n}{N-1} \min \left\{ \max \left( 0, \frac{v - p_j}{t} \right) - \frac{1}{2} \right\}, & \text{for } x \leq \frac{1}{2}.
\end{cases} \tag{27}
\]

Because this model is far more complex than the previous model, we use numerical analysis to understand equilibrium behavior. On examining how the popularity of the reference product affects market prices, we have the following result.

Result 1: When consumer valuation is high, the prices of the reference product and non-reference products decrease with the popularity of the reference product. However, when consumer valuation is low, the results are reversed.

One might expect the price of the reference product to increase as its popularity increases. Yet, the above result shows that when consumer valuation is high, we might observe the
opposite outcome. To follow the rationale for this finding, recall that when valuation is high, consumers could gain a surplus from buying any of the available products. In this context, consistent with Proposition 2, the reference product is priced higher than a nonreference product, and the marginal consumer lies closer to the reference product. Furthermore, the sales of the reference product are lower but its margin is higher. Now as the popularity of the reference product increases, the mass of consumers on spoke $z$ increases. This motivates the reference product to cut its price so that it can sell to more of the consumers for whom it is the first-preferred product. The nonreference products also cut their prices in response.

For an illustration, consider the case where $N = 4, n = 3, t = 1, \mu = 0.1$ and $v = 3.3$. When $\alpha = 0.28$, the equilibrium prices are $p_z = 2.050, p_j = 1.978$. When $\alpha$ rises to $0.31$, $p_z$ decreases to $1.956$ and $p_j$ decreases to $1.854$. It is useful to note that in equilibrium $\frac{v-p_z}{t} = 1.250$, and $\frac{v-p_j}{t} = 1.322$, implying that all consumers can gain a surplus by buying any of the available products.

When consumer valuation is low, we observe a different equilibrium outcome. In keeping with Proposition 1, the reference product is lower priced compared to a nonreference product. Consequently, the marginal consumer is closer to a nonreference product. In this case, the reference product’s margin is lower but it sells more. As the popularity of the reference product increases, the reference product finds that it can raise its price without losing too many sales. This in turn softens price competition. For a concrete example, consider a market where $N = 4, n = 3, t = 1, \lambda = 2, v = 1.3, \alpha = 0.28$. The equilibrium prices are: $p_z = 0.530$ and $p_j = 0.563$. When the size of $\alpha$ grows to $0.31$, $p_z$ increases to $0.568$ and $p_j$ increases to $0.573$. In equilibrium, in the low valuation region we should have $\frac{v-p_z}{t} > \frac{v-p_j}{t} > 1$, suggesting that some consumers might not find it profitable to buy any of the available products. Accordingly in this example we have $\frac{v-p_z}{t} = 0.770, \frac{v-p_j}{t} = 0.737$. Next, on comparing the profits of the popular reference product with those of a nonreference product, we have the following result.

**Result 2:** The relative profitability of the reference product increases with its popularity. Moreover, when consumer valuation is high, the reference product becomes more profitable than a nonreference product at an even lower threshold of $\mu$.

According to Proposition 4, the reference product always earns more profits than a non-
reference product when consumer valuation is low. In keeping with Proposition 4, when consumer valuation is low the popular reference product earns more profits. To see this, note that when \( N = 4, n = 3, t = 1, \lambda = 2, v = 1.3, \alpha = 0.28 \), we have \( \pi_z = 0.180 > \pi_j = 0.152 \). When \( \alpha \) grows to 0.31, \( \pi_z \) increases to 0.198 but \( \pi_j \) decreases to 0.151.

When consumer valuation is high, the reference product is more profitable than a non-reference product only if \( \mu \) is above a threshold. Proposition 4 stipulates that the reference products will earn more profits only when \( \mu > \frac{2n-1}{(n-1)(n-2)} \). When we take into account the greater popularity of the reference product, the reference product starts to earn more profits than a nonreference product at an even lower threshold of \( \mu \). For example, when \( N = 4, n = 3, t = 1, \mu = 0.1, v = 3.3, \) and \( \alpha = 0.28 \), we find that \( \pi_z = 0.713 > \pi_j = 0.645 \). Note that \( \mu > \frac{2n-1}{(n-1)(n-2)} = \frac{5}{2} \), which is more than \( \mu = 0.1 \). When \( \alpha \) grows to 0.31, \( \pi_z \) increases to 0.719 but \( \pi_j \) decreases to 0.586.

6. CONCLUSION

The purpose of the paper is to understand how reference-dependent utility affects price competition in a horizontally differentiated market where consumers’ tastes are very diverse and firms offer a wide variety of products. Toward this goal, we incorporate the notion of reference dependence in a spokes model and examine its implications for price competition. Our analysis shows that the rank order of the equilibrium prices of the reference product and a nonreference product depends on consumer valuation. The reference product charges a lower price than a nonreference product when consumer valuation is low, but the opposite holds when consumer valuation is high. One may wonder whether this is a peculiarity of the spokes model. It is not. In the absence of reference dependence, if \( N = n \geq 3 \) the spokes model has a unique equilibrium where all the firms charge the same price, namely \( t \), implying that our results are driven by reference dependence.

When consumer valuation is low, loss aversion on price is the dominant force operating in the market, and the reference product takes advantage of it by charging a lower price and garnering more market share. A nonreference product cannot gain by undercutting the reference product’s price because the resulting loss in revenue exceeds the gain in sales. On the other hand, when consumer valuation is high, loss aversion on taste comes to play an
important role. This is because the marginal consumer is on the spoke where the reference product resides and thus gains a higher match value from the reference product, not a nonreference product. Moreover, buying a nonreference product does not induce any psychological loss on price because the reference product is priced higher.

Turning attention to profits, the reference product always earns more profits than a nonreference product by charging a lower price when consumer valuation is low. The lower price of the reference product helps it to gain sales from each of the nonreference products, and the total gain in sales more than offsets the reduction in margin. On the other hand, when consumer valuation is high, the reference product can earn higher profits than a nonreference product by charging a higher price only if loss aversion on match quality is above a threshold. Only then can the loss in sales be offset by the higher margin. In many product categories the reference product is the most preferred product of a large fraction of consumers. In such instances, the reference product will yield more profits than a nonreference product at an even lower threshold of loss aversion.

Our analysis also sheds light on how market coverage could moderate the effect of diversity in consumers’ tastes on equilibrium profits. In a partially covered market, as consumers’ tastes become more diverse, the fraction of its sales to the monopoly segments increases and that to the competitive segments declines. In such a situation, if consumer valuation is low, competing firms are more motivated to reduce their prices so that they can better cater to low valuation consumers in the monopoly segment. But if consumer valuation is high, firms could raise their prices and earn more profits from the high valuation consumers in the monopoly segment.

There are several avenues for further research. While our analysis focuses on a horizontally differentiated market, reference-dependent preference is also pertinent for a vertically differentiated market. For instance, while the iPhone and high-end android phones could be viewed as horizontally differentiated products, the iPhone is certainly superior in quality to many mid and low-end phones. Future research can explore the strategic implications of reference-dependent utility in such multi-tiered markets by using a model of vertical differentiation. In retail stores, we observe multi-tiered store brands, and reference-dependent utility may have a bearing for such markets as well (e.g., Amaldoss and Shin 2015). Some
product categories may have multiple reference products because of heterogeneity across consumers. For example, Coke and Pepsi could be the reference products in the carbonated soft drink category. It would be interesting to examine the effect of reference dependence in such markets.

References


