Sustainable Shadow Banking

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Abstract

Commercial banks are subject to regulation that restricts their investments. When banks are concerned for their reputation, however, they could self-regulate and invest more efficiently. Hence, a shadow banking that arises to avoid regulation has the potential to improve welfare. Still, the strength of reputation concerns depends on future economic prospects and may suddenly disappear, generating a collapse of shadow banking and a return to traditional banking, with a decline in welfare. I discuss a combination of traditional regulation and cross reputation subsidization that enhances shadow banking and makes it more sustainable.

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1 Introduction

Banks’ activities are regulated to prevent excessive risk-taking and to provide a safety net to investors and depositors. One of those regulations involves capital requirements, which impose a minimum level of capital that banks should hold as a fraction of their risk-weighted assets. Banks can relax this constraint either by holding more capital as a fraction of total assets, or by holding a smaller fraction of assets that are broadly classified as risky by regulators (such as corporate debt, real estate or emerging market debt). Restricting risk-taking, however, comes at the cost of forcing banks to give up opportunities that they identify as superior and relatively safe but belong to a class of investments that regulators, usually with less knowledge of the market, deem as risky.1

In the run up to the 2007-09 financial crisis in the United States, banks increasingly devised securitization methods to get around capital requirements without reducing investments in risky assets. An example was the sponsoring of asset backed commercial paper (ABCP) conduits. These conduits are special purpose vehicles (SPV) designed to purchase and hold long-term assets from banks by issuing short-term ABCPs to outside investors. Since these assets are classified off-balance sheet once held by SPVs, they are not considered for the purpose of computing capital requirements. Indeed, Gilliam (2005) computed that regulatory charges for conduit assets were 90% lower than regulatory charges for on-balance sheet assets. Furthermore, Acharya, Schnabl, and Suarez (2013) show that banks using ABCP more intensively before the crisis were those more heavily constrained by regulation, suggesting these conduits were effective in avoiding regulatory pressures.2

This movement away from traditional banking into so called shadow banking – intermediation usually associated with traditional banking, but that runs in the “shadow” of regulators – was documented by Poznar et al. (2012), “At the eve of the financial crisis, the volume of credit intermediated by the shadow banking system was close to $20 trillion, or nearly twice as large as the volume of credit intermediated by the

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1For a summary of the weights that new Basel III regulations assign to broad asset classes see www.bis.org/bcbs/basel3.htm.
2Interestingly, Acharya and Schnabl (2010) also note that Spain and Portugal are the only European countries that impose the same regulatory capital requirements for both assets on balance sheet and assets on ABCP conduits. Consistently with the regulatory arbitrage motive, banks in these countries do not sponsor ABCP conduits.
traditional banking system at roughly $11 trillion. Today, the comparable figures are $16 and $13 trillion, respectively.”

More specifically, Acharya, Schnabl, and Suarez (2013) show that ABCP “grew from $ 650 billion in January 2004 to $ 1.3 trillion in July 2007. At that time, ABCP was the largest money market instrument in the United States. For comparison, the second largest instrument was Treasury Bills with about $ 940 billion outstanding.” This large increase came to a sudden halt on August 9, 2007, when BNP Paribas suspended withdrawals from three funds invested in mortgage backed securities. “The interest rate spread of overnight ABCP over the Federal Funds rate increased from 10 basis points to 150 basis points within one day of the BNP Paribas announcement. Subsequently, ...ABCP outstanding dropped from $1.3 trillion in July 2007 to $833 billion in December 2007.”

To compensate investors for participating in shadow banking without the safety net that regulations provide, banks offered explicit guarantees to repay maturing ABCP at par. However, investors still have to be confident that banks do not take excessive risks when originating the assets and that they can honor such guarantees. Why do investors agree on participating in shadow banking if they understand that banks are trying to avoid regulation that provides a safety net against excessive risk-taking? A potential answer is that indeed regulation and capital requirements are useless. However, if this were true, why would investors run from shadow to traditional banking when they become concerned about the quality of banks’ assets?

I argue that reputation concerns lie at the heart of both the growth and the fragility of shadow banking. Shadow banking spurs as long as outside investors believe that capital requirements are not critical to guarantee the quality of banks’ assets, since reputation concerns self-discipline banks’ behavior. When bad news about the future arise reputation becomes less valuable and investors stop believing in the self-discipline of banks, moving their funds to a less efficient, but safer, traditional banking.

A narrative of the recent crisis postulates that, since financial institutions using shadow banking were unregulated, there was excessive risk-taking that led to low quality as-

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3This surprising growth and posterior collapse of shadow banking is not unprecedented. During the late 19th century and early 20th century all banking was effectively “shadow” given the absence of regulation, and experienced similar changes – after large expansions of banking activities, from time to time investors suddenly became concerned about the credit quality and liquidation values of the collateral used to back financial assets, coordinating on bank runs, such as the large run of 1907.
sets and an insolvency crisis. Instead, I argue that the collapse of shadow banking was not the result of realized excessive risk-taking previous to the crisis but the arrival of news that led to expectations of future excessive risk-taking. This change led investors to move from an unregulated shadow banking that was operating as if it were regulated, to traditional banking regulated by the government. Indeed, Park (2011) shows that there is no evidence of excessive risk-taking previous to the crisis.\footnote{Park calculates the realized principal losses on the almost 2 trillions dollars of AAA/Aaa-rated subprime bonds issued between 2004 and 2007 to be just 17 basis points as of February 2011.}

In the model, banks have capital but also need to borrow from outside investors. Banks can invest these funds in safe assets or risky assets. Safe assets enjoy a relatively high probability of success but pay a moderate return when succeeding. Risky assets come in two types, which are only observable by bankers, not by governments or lenders: \textit{Inferior risky assets} that pay a high return in case of success but have a low probability of success and \textit{superior risky assets} that also pay a high return in case of success, but have the same high probability of success as safe assets. I assume a planner would like banks to invest in risky assets if the risky asset is superior and in safe assets if the risky asset is inferior. However, when accounting for the cost of funding, banks have incentives to take excessive risk, always investing in risky assets regardless of their type. The critical tension this setting captures is that bankers have the knowledge to invest efficiently but not the incentives, while governments have the incentives but not the knowledge. Then, governments would rather have bankers investing in known safe assets than in unknown risky assets, even at the cost of foregoing profitable investment opportunities.

Two mechanisms have the potential to prevent excessive risk-taking. One is \textit{government based}; regulation in the form of capital requirements that impose restrictions on the fraction of risky assets per unit of capital. If the government cannot identify the type of risky asset, this regulation prevents investments in inferior risky assets, but also in superior risky assets. The other is \textit{market based}; self-discipline sustained by reputation concerns, which prevent investments in inferior risky assets, but not in superior risky assets. Self-discipline, however, is fragile.

When the future looks bright, with good expected business opportunities, building and maintaining reputation is valuable, inducing banks to invest optimally. However, when future prospects are poor, reputation is not that valuable, increasing the
incentives of banks to take excessive risks. Since reputation concerns induce multiple equilibria, non-obvious changes in news about economic fundamentals can lead to sudden changes in banks’ behavior, with investors reacting by moving from the shadow banking back to a less efficient traditional banking.

How to enhance shadow banking? How to make it less volatile? I show that a combination of capital requirements and cross subsidization of banks – taxing those with low reputation and subsidizing those with high reputation, such that the policy budget is balanced – can stabilize shadow banking. This novel regulation reduces even further the reputation concerns of banks with already low concerns, which is irrelevant for welfare since those banks cannot participate in shadow banking in the first place. In contrast it increases even further the reputation concerns of banks with already high concerns, making them more willing to participate in shadow banking and their self-regulation less sensitive to changes in future economic conditions, rendering shadow banking less fragile.

This result is relevant from a policy perspective. Under standard banking regulations the economy is more stable but achieves less output in expectation. When banks can operate using shadow banking, the economy is more volatile but achieves a higher output in expectation. In my model shadow banking always dominates because I assume risk neutral agents, but risk aversion would introduce a trade-off between output level and output volatility. Still, even if parameters are such that traditional banking is preferred from a welfare perspective, banks would always prefer ways around regulations by creating shadow banking activities. The novel regulation I discuss in this paper both increases expected output and reduces volatility when shadow banking is an unavoidable possibility.

Relation with the literature. The idea that regulatory constraints may trigger financial innovation has a long-standing tradition in the literature.\footnote{See, for example, Silber (1983) and Miller (1986)} However, since the implementation of Basel accords, regulatory arbitrage has attracted a lot of renewed attention. Stein (2010) highlights as one of the main forces behind securitization the circumvention of capital and other regulatory requirements, while Acharya, Schnabl, and Suarez (2013) document this motive empirically. In this paper I rationalize the rise and collapse of shadow banking by the rise and collapse of self-discipline, an efficient alternative to government regulation driven by fragile reputation concerns.
My rationalization of shadow banking complements other explanations that focus on the risk-sharing properties of securitization. Gennaioli, Shleifer, and Vishny (2012) argue that an increase in investors’ wealth drives up securitization, introducing fragility because banks become interconnected and more exposed to systemic risk. As in my paper securitization is also welfare improving, but in contrast to my paper crises do not happen under rational expectations but only when agents neglect systemic risks.

The paper also complements the view of Adrian and Shin (2009) and Shin (2009), who argue that securitization facilitates reaching investors and accessing their funds, increasing credit supply. In their work, as balance sheets expand the needs for new borrowers lead banks to lower lending standards, reducing asset quality and leaving the system in a fragile position to face downturns. In my paper, the lower quality of assets from inefficient risk-shifting (more relaxed lending standards, for example) arises from a collapse in self-regulation and not from a higher demand for funds.

Even in the presence of recent heated debates on how to regulate financial markets, the literature that formally studies regulatory arbitrage and its link with shadow banking is still scarce. An exception is Plantin (2014) who argues that “relaxing capital requirements for traditional banks so as to shrink shadow activity may be more desirable than tightening them.” In his paper, governments can implement optimal regulations, and then shadow banking arises to reduce welfare. In my paper, governments are constrained on their knowledge and how much they can improve welfare with regulation. Then the rise of shadow banking can indeed be beneficial.

Gorton and Metrick (2010) structure their proposal to regulate shadow banking around the idea that securitization arises because it is bankruptcy remote. Ricks (2010) also proposes to extend the safety net of public insurance to shadow banking to reduce its fragility. However, as Adrian and Ashcraft (2012) extensively document, regulation has persistently failed in stabilizing shadow banking. I argue this view of regulation may be misleading since banks can always find ways around regulation when self-regulation becomes feasible, and it is indeed efficient for them to do so.

Consistently with the view in this paper, Hanson, Kashyap, and Stein (2011) argue that recent proposals to heighten capital requirements for traditional banks may trigger even more regulatory arbitrage, thereby inducing a large migration of banking

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6See also Harris, Opp, and Opp (2014), where banks compete with shadow banking institutions rather than engaging in shadow banking activities themselves."
activities towards the unregulated shadow banking system. I propose to focus on “carrots” rather than “sticks” to discipline banks, with policy interventions that enhance the reputation discipline by cross-subsidizing banks, reducing the fragility of securitization and still maintaining its efficiency.\footnote{Atkeson, Hellwig, and Ordonez (2012) also discuss the benefits of cross-subsidization in a general equilibrium environment with free entry of reputationally concerned firms.}

Finally, even though this paper rationalizes the rise and collapse of shadow banking characterized by conduits with explicit guarantees and investors concerned about the quality of those guarantees, an important fraction of securities were not guaranteed at all. For those financial instruments, the investors’ main concern is the incentives of sponsoring banks to cover securities in distress with balance-sheet assets, even without being legally bound to do it (that is, implicit guarantees). According to Acharya, Schnabl, and Suarez (2013), among all ABCP outstanding in 2007, 10% were barely guaranteed – extendible and SIV weak guarantees – in commercial banks, 50% in structured finance companies, and almost 75% in mortgage originators.

In Ordonez (2013a), I show that conduits that do not have explicit guarantees can also be sustained by investors’ beliefs that banks concerned for their reputation have incentives to take securities in distress back to their balance-sheet. With these conduits, banks not only avoid regulation but also, given that there are no explicit guarantees, save on potential bankruptcy costs. Gorton and Souleles (2006) also propose that securitization arises as an implicit collusion between banks and investors to save on bankruptcy costs. However, in their paper there is no discussion on how the system sustains such collusion or how it can collapse.

In the next section I introduce a model that compares traditional and shadow banking, highlighting that shadow banking instruments, such as securitization, provide a more efficient, but fragile, alternative to traditional banking. I also discuss how banks choose between these two banking systems based on their reputation. In Section 3, I discuss a novel regulation to enhance and stabilize shadow banking. In Section 4, I illustrate the results and the dynamics with a numerical example. In Section 5, I make some concluding remarks.
2 Model

2.1 Description

Consider a two-period economy with a continuum of banks and lenders, and a government. There are two types of banks: Good banks (G) care about their future (low discount rate), while bad banks (B) do not care about their future (infinite discount). Banks observe their own type. We define reputation of a bank $\phi$ as the probability that the bank is of type $G$.

In the first period, each bank has a single unit of capital and can invest in two assets. The first asset, that I call a “new asset”, pays $x$ with probability $p_x$ in the second period, and 0 otherwise. The second asset, that I call an “old asset”, comes in two available types: “Safe assets” that pay $y_s$ with probability $p_s$ in the second period, and 0 otherwise, and “risky assets” that pay $y_r > y_s$ in case of success. With probability $\alpha$ the risky asset is superior and succeeds with probability $p_s$. With probability $1 - \alpha$ the risky asset is inferior and succeed with probability $p_r < p_s$. Banks observe the risky assets’ types, but lenders and governments do not.

I assume the investment in each asset requires one unit of capital. Hence, if a bank wants to invest in both new and old assets it has to borrow an additional unit of capital, at an endogenous rate $R$, from infinitely many, short-lived, risk-neutral, and perfectly competitive lenders, whose outside option generates a risk free return normalized to 0.

To make the model interesting, I introduce the following assumptions on payoffs.

Assumption 1 Assets’ Payoffs

1. $p_x x > p_s y_r > p_s y_s > p_r y_r > 1$ (New assets pay in expectation more than old assets. Superior risky old assets pay in expectation more than safe old assets, which pay in expectation more than inferior risky old assets. All investments are ex-ante efficient).

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8 This is a stylized way to introduce heterogeneity in quality. Alternatively, bad banks can expect to perform poorly and leave the banking industry more likely than good banks.

9 I denote these assets as new and old respectively to convey the idea that it is easier to identify types for assets that have been traded for some time. It will become clear that this distinction just make equations simpler to interpret because the new asset just drops out.

10 In Section 4 I show a numerical example where all these assumptions are satisfied.
2. $p_s y_s > \alpha p_r y_r + (1 - \alpha) p_r y_r$ (Absent information about risky assets’ types, it is optimal to invest in safe assets).

3. $y_r > y_s > R$ and $x > R$ (Successful assets are enough to repay loans, where $R$ is endogenous but expressed in terms of primitives).

4. $p_s(y_r - \tilde{R}) > p_r(y_r - \tilde{R}) > p_s(y_s - \tilde{R})$ where $\tilde{R} = (1 - p_x)R$ is the interest rate banks have to pay with proceedings from the old asset, when the new asset fails (with probability $1 - p_x$). (Risk-shifting. Banks always prefer to invest in risky assets).

In short, these assumptions imply that all investments are ex-ante efficient. Banks always invest in the new asset first and has to choose between a safe or risky old asset later because they cannot invest in more than two assets.\textsuperscript{11} With information about risky assets’ types, it is efficient to invest in risky assets when risky assets are superior and in safe assets when risky assets are inferior. Absent information about risky assets’ types, it is efficient to invest in safe assets. However, when one account for the cost of funding, banks prefer to always invest in risky assets.

**Financing choices:** To raise funds, banks have to choose between using traditional banking or shadow banking.

The use of traditional banking implies issuing debt, such as collecting demand deposits in the case of commercial banks. I assume banks issue debt instead of equity because, by its liquidity properties, debt is one of the main “services” that banks provide, indeed defining their existence as discussed by Gorton and Pennacchi (1990), Dang, Gorton, and Holmström (2013) and Dang et al. (2014). However, since debt also has well-known risk-shifting problems when there is limited liability (banks tend to take excessive risk because they obtain the gains of upside realizations and transfer to debt holders the losses of downside realizations), banks are subject to regulations and capital requirements that introduce restrictions on how they invest.

To be more precise about how risk-weighted capital requirements restrict investments, assume the weight assigned to new assets is $\omega_n$, to old safe assets $\omega_s$ and to old risky assets $\omega_r$. Regulators define the weight for each asset, for example, by using its failure probability (for instance, $\omega_n = 1 - p_x$). Since governments cannot distinguish

\textsuperscript{11}This assumption captures the need of banks to choose their risk exposure and can be rationalized by limits of banks to manage assets, or limits of investors’ wealth.
between risky assets, and risky assets are relatively less likely to repay than safe assets, then \( \omega_s < \omega_r \). This also implies that weighted safe assets are less than weighted risky assets, \( \omega_s y_s < \omega_r y_r \), since \( y_s < y_r \) by assumption 1.2.

Since capital is assumed to be 1, the fraction of capital over risk-weighted assets is \( \frac{1}{\omega_n x + \omega_s y_s} \) if the bank invests in safe old assets and \( \frac{1}{\omega_n x + \omega_r y_r} \) if the bank invests in risky old assets. Regulators can prevent excessive risk-taking by setting the weights and the minimum requirement (that I denote by \( \chi \)) such that

\[
\frac{1}{\omega_n x + \omega_s y_s} > \chi > \frac{1}{\omega_n x + \omega_r y_r}
\]

This regulation then is isomorphic to regulators imposing investment in safe assets.\(^\text{12}\)

The use of shadow banking implies using securitization. We model one of the most commonly used forms of securitization, which is the sponsoring of ABCP conduits. In the model this is captured by the bank sponsoring a SPV, which purchases the new asset from the bank by issuing short-term ABCP that sells to the lenders. By selling the new asset and making it off-balance sheet, banks can both fulfill capital requirements and invest in either safe or risky assets without restrictions as long as

\[
\frac{1}{\omega_s y_s} > \frac{1}{\omega_r y_r} > \chi
\]

To attract investors to acquire ABCP, banks can issue explicit guarantees to cover securities in distress (in this case a failing new asset) with successful assets on the balance sheet (in this case a successful old asset). However, not being subject to regulation, banks cannot guarantee to investors the safety of old assets in the balance sheet at origination and, as a consequence, the value of those guarantees.\(^\text{13}\)

In summary, with this setting in mind and to simplify the exposition, we simply denote the use of traditional banking as issuing debt, which imposes the restriction of investing in the safe old asset and the use of shadow banking as securitizing, which does not restrict banks on investing in safe or risky old assets. In essence, the decision

\(^\text{12}\)Banks could also relax the constraint and invest in risky assets by raising more equity (increasing the numerator). However by managing their portfolio and using shadow banking instead banks are able to invest each unit of capital without constraints.

\(^\text{13}\)A companion paper analyzes why reputation concerns are critical for the use of other type of securitization, such as structured investment vehicles (SIV), that is not guaranteed. In that case, the rationale for securitization is not only avoiding regulation but also costly bankruptcy procedures.
between using traditional or shadow banking boils down to a decision between using a system where the regulator chooses the quality of banks’ assets or a system where banks choose the quality of banks’ assets, such that lenders react accordingly.

Banks invest optimally when buying risky assets only when they are superior. Without regulation, banks have incentives to invest only in risky assets, which we call excessive risk-taking. With regulation banks are constrained to invest only in safe assets. By assumption 1, optimal investments dominate regulation, which at the time dominates excessive risk-taking.

Finally, upon repayments, banks obtain a positive continuation value \( V(\phi, \theta) \) at the end of the second period, which is an exogenous function monotonically increasing both in reputation \( \phi \) and in a unidimensional aggregate fundamental \( \theta \). While banks \( G \) discount this continuation value with a factor \( \beta > 0 \), banks \( B \) do not take this continuation value into account (their \( \beta \) is 0). The fundamental \( \theta \) represents aggregate demand, economic conditions, housing prices, or in general any other variable that positively affects the expected prospects of banks after the second period. I assume this fundamental is drawn from a known normal distribution with mean \( \mu \) and variance \( \frac{1}{\gamma_\theta} \) (i.e., precision \( \gamma_\theta \)).\(^{14}\) Upon default, regardless of the financing choice, the continuation value is zero. This can be interpreted in the repeated game as the bank being liquidated and unable to re-enter the industry to raise new funds from investors or depositors.\(^{15}\)

**Timing:** Summarizing the timing in both periods

- **Period 1:** All agents know the distribution \( \theta \sim N(\mu, \frac{1}{\gamma_\theta}) \) of fundamentals. A bank of type \( i \in \{B,G\} \) and reputation \( \phi \) chooses whether to finance the new asset by issuing (regulated) debt or (unregulated) securities. All agents observe the fundamental \( \theta \) and only banks observe the type of the risky asset. If banks issue debt, they have to invest in safe assets. If banks issue securities, they choose whether to invest in a risky or a safe asset.

\(^{14}\)For expositional reasons, I assume \( V(\phi, \theta) \) is exogenous. It is easy to endogeneize the continuation value as a positive function of \( \phi \) in a full fledged repeated game (since I will show that endogenous interest rates decrease with \( \phi \)) and to show that continuation values are positive under limited liability. These extensions are cumbersome and unnecessary to illustrate the main points of the paper. For an application of how to endogeneize value functions, see Ordonez (2013b).

\(^{15}\)This extreme assumption, which imposes a heavy punishment from defaulting, is just a normalization that simplifies the exposition.
• **Period 2:** Asset payoffs are realized and observed by all agents. The risky asset type, however, remains unknown to lenders and regulators.\textsuperscript{16} If the bank does not repay (both assets fail), it is liquidated and disappears with a continuation value of 0. If the bank repays, it continues, its reputation is updated from \( \phi \) to \( \phi' \) according to Bayes’ rule and it obtains a continuation value \( V(\phi', \theta) \).

In what follows, I first characterize separately the payoffs from debt and from securitization for a bank of a given type \( i \in \{B, G\} \) and reputation \( \phi \). Then, I characterize the optimal financing decision of banks of different types and reputations.

## 2.2 Traditional Banking

Since lenders are competitive and the risk-free rate is zero, interest rates \( R \) equalize the expected repayment with the size of the loan, normalized to 1. Since banks are forced to invest in safe assets, \( \hat{p}_D = p_x + (1 - p_x)p_s \) can be defined as the probability of loan repayment, where \( p_x \) is the probability that the new asset succeeds and \( p_s \) is the probability that the safe asset succeeds. The face value of debt (in this case, also the interest rate) is the loan divided by the expected probability of repayment. Then,

\[
R_D = \frac{1}{\hat{p}_D},
\]

Note the interest rate \( R \) is independent of banks’ type and, as a consequence, also independent of banks’ reputation, \( \phi \). After repayment, reputation is not updated because both good and bad banks choose the same assets, determining the bank’s continuation value, \( V(\phi, \theta) \).

## 2.3 Shadow Banking

In this case banks avoid regulatory pressures, and they are not restricted on old assets’ investments. By construction bad banks only invest in risky assets if not regulated,
then the expected probability a bad bank ($B$) repays an issued security is

\[ \hat{p}_B = p_x + (1 - p_x)[\alpha p_s + (1 - \alpha)p_r] \]
\[ = p_x + (1 - p_x)[p_s - (1 - \alpha)(p_s - p_r)] < \hat{p}_D. \]

Bad banks repay when the new asset is successful (with probability $p_x$), and when the new asset fails, the guarantee imposes repayment using the assets on the balance sheet. Since these are risky assets, their probability of success is $p_s$ if the asset is superior (with probability $\alpha$) and $p_r$ if the asset is inferior (with probability $1 - \alpha$).

Similarly, the expected probability a good bank ($G$) repays an issued security is

\[ \hat{p}_G(\hat{\tau}) = p_x + (1 - p_x)[\alpha p_s + (1 - \alpha)(p_s\tau + p_r(1 - \tau))] \]
\[ = \hat{p}_B + \hat{\tau}(1 - p_x)(1 - \alpha)(p_s - p_r) \leq \hat{p}_D, \]

where the strategy $\tau \in [0, 1]$ is the probability good banks invest optimally (invest in safe assets when risky assets are inferior) and $\hat{\tau}$ is the lenders’ beliefs about good banks’ strategies. When $\hat{\tau} = 1$, and lenders believe good banks invest optimally, then $\hat{p}_G(\hat{\tau} = 1) = \hat{p}_D$. In contrast, when $\hat{\tau} = 0$, then $\hat{p}_G(\hat{\tau} = 1) = \hat{p}_B$.

The price of securities can then be expressed as an interest rate,

\[ R_S(\phi|\hat{\tau}) = \frac{1}{\phi\hat{p}_G(\hat{\tau}) + (1 - \phi)\hat{p}_B} = \frac{1}{\hat{p}_B + \phi\hat{\tau}(1 - p_x)(1 - \alpha)(p_s - p_r)} \geq R_D. \quad (2) \]

The only situation in which $R_S = R_D$ is when $\phi = 1$ and $\hat{\tau} = 1$. In this extreme case it is believed the bank is good for sure and good banks always invest optimally. In all other cases, raising funds issuing securities is more expensive than issuing debt.

Reputation updating also depends on beliefs, $\hat{\tau}$. Using Bayes’ rule,

\[ \phi'(\phi|\hat{\tau}) = \frac{\hat{p}_G(\hat{\tau})\phi}{\phi\hat{p}_G(\hat{\tau}) + (1 - \phi)\hat{p}_B}. \]

As shown in Figure 1, $\phi'(\phi|\hat{\tau})$ increases with $\hat{\tau}$ for a given $\phi$. Intuitively, if lenders believe that good banks invest optimally, then they expect that good banks are more likely to repay than bad banks, who only invest in risky assets. Given these beliefs, lenders will revise reputation up when they observe a bank repaying securities and
covering guarantees. In contrast, if lenders expect that good banks never invest optimally, they expect that good banks repay with the same probability as bad banks, not revising reputation when observing repayment of a security.

Figure 1: Reputation Updating

Expected profits for good banks with reputation $\phi$ following strategy $\tau$, conditional on lenders believing good banks with reputation $\phi$ follow strategy $\hat{\tau}$ are

$$U_G^S(\phi, \theta, \tau | \hat{\tau}) = p_x x + \alpha p_s y_r + (1 - \alpha)[\tau p_s y_s + (1 - \tau)p_r y_r] + \hat{\mu}_G(\tau) [\beta V(\phi'(\phi|\hat{\tau}), \theta) - R_S(\phi|\hat{\tau})].$$

Good banks invest optimally ($\tau = 1$) given beliefs $\hat{\tau}$, if

$$\Delta(\phi, \theta | \hat{\tau}) = U_G^S(\phi, \theta, \tau = 1 | \hat{\tau}) - U_G^S(\phi, \theta, \tau = 0 | \hat{\tau}) > 0,$$

which can be rewritten as

$$\Delta(\phi, \theta | \hat{\tau}) = (1 - \alpha)(p_s y_s - p_r y_r) + (1 - p_x)(1 - \alpha)(p_s - p_r)[\beta V(\phi', \theta | \hat{\tau}) - R_S(\phi | \hat{\tau})] > 0.$$

**Definition 1** A reputation equilibrium is one in which good banks invest optimally, and beliefs are consistent, $\tau = \hat{\tau} = 1$. A non-reputation equilibrium is one in which good banks take excessive risk (always invest in the risky asset), and beliefs are consistent, $\tau = \hat{\tau} = 0$. 

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Defining the short-term social loss of investing in an inferior risky project as

$$\Psi \equiv \frac{pr_yy_r - ps_y}{(1-ps)(ps-pr)} < 0,$$

the sufficient condition for a reputation equilibrium is

$$\beta V(\phi', \theta | \hat{\tau} = 1) \geq \Psi + R_S(\phi | \hat{\tau} = 1) > 0.$$ \hspace{1cm} (3)

where $$\Psi + R_S(\phi | \hat{\tau}) = \frac{ps_y(y_r - (1-ps)R_S) - pr_y(y_r - (1-ps)R_S)}{(1-ps)(ps-pr)} > 0$$ represents the short-term private gains of investing in inferior risky assets and it is positive for $$\hat{\tau} = 1$$, which is the lowest interest rate in equilibrium, by Assumption 1.4.

In contrast, the condition for a non-reputation equilibrium, in which $$\tau = \hat{\tau} = 0$$, is

$$\beta V(\phi', \theta | \hat{\tau} = 0) \leq \Psi + R_S(\phi | \hat{\tau} = 0).$$ \hspace{1cm} (4)

In what follows, I describe a potential multiplicity of equilibria and refine the set of equilibria using global games techniques. Then, using the unique equilibrium obtained from the refinement, I characterize the conditions under which different financing decisions are implemented.

### 2.3.1 Multiplicity with complete information

Since continuation values are monotonically increasing in posteriors $$\phi'$$, which are monotonically increasing in $$\hat{\tau}$$, then $$V(\phi', \theta | \hat{\tau} = 1) > V(\phi', \theta | \hat{\tau} = 0)$$. Also, since $$\hat{\rho}_G(\hat{\tau} = 1) > \hat{\rho}_G(\hat{\tau} = 0)$$, then $$R_S(\phi | \hat{\tau} = 1) < R_S(\phi | \hat{\tau} = 0)$$. Combining these inequalities with equilibrium conditions (3) and (4), there are values of $$\theta$$ under which reputation and non-reputation equilibria coexist. Fundamentals are not only useful to characterize multiplicity in an environment with changing conditions, but are also key in selecting a unique equilibrium using global games techniques, by assuming that agents do not observe $$\theta$$ perfectly, but ”almost” perfectly.

Good banks invest optimally when the expected gains from reputation are large enough. Since these gains increase with fundamentals $$\theta$$, I focus on cutoff strategies,

$$\tau(\phi, \theta) = \begin{cases} 
1 & \text{if } \theta > \theta^*(\phi) \\
[0, 1] & \text{if } \theta = \theta^*(\phi) \\
0 & \text{if } \theta < \theta^*(\phi)
\end{cases}$$
Given these strategies I redefine good banks’ repayment probabilities as

$$p_G(\theta^*) = \hat{p}_B + (1 - \mathcal{N}(\hat{\theta}^*)) (1 - p_x) (1 - \alpha) (p_s - p_r),$$

where $\hat{\theta}^*$ is the cutoff lenders believe good banks will follow and $\mathcal{N}(\hat{\theta}^*)$ is the ex-ante expectation that $\theta < \theta^*(\phi)$ when the expectation of $\theta$ is $\mu$. Then,

$$R_S(\phi|\theta^*) = \frac{1}{\hat{p}_B + \phi (1 - \mathcal{N}(\hat{\theta}^*)) (1 - p_x) (1 - \alpha) (p_s - p_r)} \quad (5)$$

If $\hat{\theta}^* = -\infty$, then $p_G(\hat{\theta}^* = -\infty) = \hat{p}_B + (1 - \alpha) (p_s - p_r)$ and $R_S(\phi|\hat{\theta}^* = -\infty) = \frac{1}{\hat{p}_B + \phi (1 - p_x) (1 - \alpha) (p_s - p_r)}$. If good banks are believed to always invest optimally, interest rates are lower the higher the reputation of the sponsor. In contrast, if $\hat{\theta}^* = \infty$ and $p_G(\hat{\theta}^* = \infty) = \hat{p}_B$, then $R_S(\phi|\hat{\theta}^* = \infty) = \frac{1}{\hat{p}_B}$. If good banks are believed to always invest in risky assets, then default probabilities are the same for good and bad banks, and banks pay the highest possible interest rates, independently of their reputation.

In this setting there are two possible sources of multiplicity. First, interest rates can possibly generate a finite number of equilibria. It is straightforward to select a unique equilibrium in this situation as in Stiglitz and Weiss (1981), assuming Bertrand competition in which lenders first offer a rate and then banks choose the best offer. Assume for example there are three possible interest rates in equilibrium and all lenders charge the highest rate. In this case there are incentives for a single lender to deviate, offering a lower rate consistent with a different equilibrium, attracting banks and still breaking even. Then, lenders that effectively provide loans are the optimistic ones. This refinement rationalizes as the unique equilibrium the one with the lowest rate.

The second source of multiplicity, reputation formation, is more difficult to deal with since it always generates a continuum of multiple equilibria. Can we still apply the selection mechanism proposed by Stiglitz and Weiss (1981)? Yes, but only if the lenders who update reputation are the same than those who provide loans. Only in this uninteresting case, in which a bank only obtains financing from a single lender all its life, and the perception of other market agents does not matter, is there not a meaningful complementarity problem from reputation formation.

However, if the lenders who set interest rates in the current period are different to the lenders who provide funds in following periods (or at least there is some chance
lenders are not the same, which is a realistic assumption for depositors in commercial banks, interest rates cannot be used to select an equilibrium. Assume again all lenders charge a high rate and then good banks prefer to invest only in risky assets. A single lender does not have incentive to deviate and charge a lower interest rate (as opposed to the Bertrand intuition) because the bank taking its loan still would not be induced to repay, knowing that future lenders will likely not update its reputation. Hence, even when Bertrand competition can solve multiplicity generated by the first source, it cannot solve the multiplicity created by complementarity in reputation formation.

In what follows, I assume the first source of multiplicity is not an issue, so I can focus on the more interesting multiplicity created by reputation formation. First, I assume that, fixing a belief $\hat{\tau}$ for all fundamentals $\theta$, there is a unique cutoff $\theta^*$ at which banks are indifferent between investing optimally or not.

**Assumption 2  Single Crossing**

Assume fixed beliefs $\hat{\tau}$ for all $\theta$. There is a unique cutoff fundamental $\theta^*$, consistent with beliefs $\tilde{\theta}^* = \theta^*$, at which banks are indifferent between investing optimally or not, such that

$$\beta V(\phi', \theta^* | \hat{\tau}) = \Psi + R_S(\phi | \tilde{\theta}^* = \theta^*).$$

This assumption is fulfilled, for example, when the variance of fundamentals is low enough, such that the ex-ante probability of default $\mathcal{N}(\tilde{\theta}^*)$, and hence interest rates $R_S$, do not change abruptly with changes in beliefs about cutoffs $\tilde{\theta}^*$.17

Now, I define a range of fundamentals for which, regardless of lenders’ beliefs, good banks with reputation $\phi$ take excessive risk and a range of fundamentals for which, regardless of lenders’ beliefs, those banks invest optimally.

**Assumption 3  Dominance Regions**

There are fundamental levels $\theta(\phi)$ under which $\beta V(\phi', \theta | \hat{\tau} = 1) < \Psi + R_S(\phi | \tilde{\theta}^* = \theta)$ and $\bar{\theta}(\phi)$ above which $\beta V(\phi', \theta | \hat{\tau} = 0) > \Psi + R_S(\phi | \tilde{\theta}^* = \theta)$.

---

17The formal derivation of a single crossing assumption in terms of the variance of fundamentals can be found in Ordonez (2013b). Here, for expositional simplicity, I just assume single crossing in terms of endogenous variables and then, in Section 4, I check the assumption is sustained in equilibrium.
For all fundamentals $\theta < \bar{\theta}$ banks take excessive risk, even if lenders believe $\hat{\tau} = 1$ and reputation suffers a lot from taking risks. Intuitively, future prospects are so poor that reputation concerns are irrelevant. Similarly, for all fundamentals $\theta > \bar{\theta}$ banks invest optimally, even if lenders believe $\hat{\tau} = 0$ and reputation does not improve from investing optimally. Here, future prospects are so good that banks are afraid of defaulting and getting a zero continuation value. Finally, $\bar{\theta}(\phi) < \bar{\theta}(\phi)$ for all $\phi$, since $\beta V(\phi', \hat{\theta}^*|\hat{\tau} = 1) - R_S(\phi|\hat{\theta}^*) > \beta V(\phi', \hat{\theta}^*|\hat{\tau} = 0) - R_S(\phi|\hat{\theta}^*)$ for all $\phi$ and $\hat{\theta}^*$.

For all $\hat{\theta}^* \in [\bar{\theta}(\phi), \bar{\theta}(\phi)]$, reputation and non-reputation equilibria coexist. In this range, good banks invest optimally when lenders believe good banks will invest optimally and take excessive risk when lenders believe good banks will take excessive risk. This implies that a fundamental $\theta$ can be defined as an equilibrium cutoff if there exists a $\hat{\tau}(\phi, \hat{\theta}^*) \in [0, 1]$ such that

$$\beta V(\phi', \hat{\theta}^*|\hat{\tau}(\hat{\theta}^*)) = \Psi + R_S(\phi|\hat{\theta}^*).$$

This multiplicity is problematic for drawing conclusions about the effects of reputation in sustaining self-regulation in the shadow banking. In the next section I follow the refinement strategy in reputational models I propose in Ordonez (2013b), relaxing the assumption of complete information about $\theta$ and selecting a unique equilibrium robust to small perturbations of information about $\theta$.

### 2.3.2 Uniqueness with incomplete information

Assume now banks $i$ and lenders $j$ observe an informative signal of the fundamental, $s_i = \theta + \epsilon_i$ where $\epsilon_i \sim N(0, \frac{1}{\gamma_s})$. Cutoff strategies are then based on signals,

$$\tau(\phi, s_i) = \begin{cases} 
1 & \text{if } s_i > s^*(\phi) \\
[0, 1] & \text{if } s_i = s^*(\phi) \\
0 & \text{if } s_i < s^*(\phi)
\end{cases}$$

The differential gains from investing optimally are now given by taking expectations about $\theta$, conditional on the prior $\mu$ and the signal $s_i$

$$E_{\theta|s_i} [\Delta(\phi, \theta|\hat{\tau}(s_i))] = (1 - \alpha) \left[ (p_s y_s - p_r y_r) + (1 - p_x)(p_s - p_r) \right] E_{\theta|s_i} [V(\phi', \theta|\hat{\tau}(s_i))] - R(\phi|\hat{\theta}^*),$$

17
where $\tilde{s}^*$ is the cutoff lenders believe banks follow. In this situation, lenders compute the interest rate to charge based on an ex-ante probability that fundamentals are smaller than $\tilde{s}^* = s^*$, such that default probability is $\mathcal{N}(s^*)$.

**Proposition 1  Unique Equilibrium.**

For $\gamma_s \to \infty$, there is a unique equilibrium in which every good bank with reputation $\phi$ invests optimally if and only if $s > s^*(\phi)$, where $s^*$ solves

$$
\beta E_{\theta|s^*} [V(\phi', \theta|\tilde{\tau}(s^*))] = \Psi + R(\phi|s^*),
$$

where $\tilde{\tau}(s^*) = 1 - \Phi(\sqrt{\gamma}(s^* - \mu))$ is the belief lenders use to update reputation when they think banks observe a signal $s^*$. Furthermore, $\gamma = \frac{\gamma_s \mu^2}{(\gamma_\theta + \gamma_s)(\gamma_\theta + 2\gamma_s)}$.

**Proof** Since $s^*(\phi)$ is the signal that makes a good bank with reputation $\phi$ indifferent between investing optimally or not, the condition that determines $s^*(\phi)$ is

$$
\beta E_{\theta|s^*} [V(\phi', \theta|\tilde{\tau}(s^*))] = \Psi + R(\phi|s^*),
$$

where $\tilde{\tau}(s_i) = 1 - Pr(E_j(\theta) < E_i(\theta)|s_i)$, this is the probability that lenders expect a fundamental $\theta$ which is smaller than the fundamental the bank expects conditional on the signal $s_i$ that the bank observes. Since at the cutoff $s^*$ banks are indifferent between investing optimally or not, $\tilde{\tau}(s^*)$ is also the probability that banks assign to lenders believing $\theta$ is such that the bank invests optimally, and hence the belief lenders use to update reputation at the cutoff $s^*$.

The updated belief of the bank about the fundamental, after observing a signal $s_i$ is

$$
E_i(\theta|s_i) = \frac{\gamma_\theta \mu + \gamma_s s_i}{\gamma_\theta + \gamma_s}.
$$

The updated distribution of the fundamentals after the bank observes the signal $s_i$ is

$$
\theta|s_i \sim \mathcal{N} \left( E_i(\theta|s_i), \frac{1}{\gamma_\theta + \gamma_s} \right),
$$

and the expected distribution of the signals that lenders observe, $s_j$, conditional on
the signal the bank does observe, $s_i$, is

$$s_j | s_i \sim \mathcal{N}\left( E_i(\theta | s_i), \frac{1}{\gamma_\theta + \gamma_s} + \frac{1}{\gamma_s} \right).$$

(7)

Hence,

$$Pr( E_j(\theta) < E_i(\theta) | s_i) = Pr \left( s_j < E_i(\theta) + \frac{\gamma_\theta}{\gamma_s} (E_i(\theta) - \mu) | s_i \right)$$

$$= \Phi \left( \sqrt{\gamma}(s_i - \mu) \right),$$

where $\gamma = \frac{\gamma_\theta \gamma_s^2}{(\gamma_\theta + \gamma_s)(\gamma_\theta + 2\gamma_s)}$.

As $\gamma_s \to \infty$, $\gamma \to 0$, then $\hat{\tau}(s_i) = \frac{1}{2}$ for all $s_i$. Hence, in the limit the unique cutoff $s^*$ is uniquely determined by $\beta E_{\theta | s^*} \left[ V(\phi', \theta | \hat{\tau}(s^*) = \frac{1}{2}) \right] = \Psi + R(\phi | s^*)$. Q.E.D.

Lenders update reputation based on their beliefs about the actions of banks, which depend on lenders’ signals. When lenders observe a signal $s_j$, they infer that the probability the bank observes a signal $s_i$ below the cutoff $s^*(\phi)$, and decides to invest optimally, is

$$\hat{\tau}(s_j) = 1 - Pr(s_i < s^* | s_j) = 1 - \Phi \left[ \sqrt{\frac{\gamma_\theta (\gamma_\theta + \gamma_s)}{\gamma_\theta + 2\gamma_s}} (s^* - \frac{\gamma_\theta \mu + \gamma_s s_j}{\gamma_\theta + \gamma_s}) \right],$$

where $\Phi$ is just the standard normal distribution from equation (7). As $\gamma_s \to \infty$, $\hat{\tau}(s_j) \to 0$ if $s_j < s^*(\phi)$ and $\hat{\tau}(s_j) \to 1$ if $s_j > s^*(\phi)$. This implies that in the limit, whenever lenders observe a signal above $s^*(\phi)$, they believe almost certainly banks invest optimally and update reputation accordingly. Similarly, whenever investors observe a signal below $s^*$, they believe almost certainly banks take excessive risk and do not update reputation.

This refinement of equilibria uncovers the fragility of reputation. A bank with reputation $\phi$ invests optimally based on a cutoff $s^*(\phi)$ and their risk-taking strategies change dramatically around that cutoff. In the next section I study this extreme sensitivity of risk-taking, which makes reputation concerns, and then shadow banking, fragile.
2.3.3 Expected Output

What is the expected output generated by the investments of a bank with a reputation level $\phi$? The answer clearly depends on the financing choice of the bank. If the bank raises funds using traditional banking, they have to invest in safe assets and the expected output is then

$$Y_D(\phi) = px + psy_s$$

If the bank raises funds using shadow banking, it chooses to invest in superior risky assets when possible (with probability $\alpha$), to invest in risky assets if risky assets are inferior and $\theta$ is low (with probability $(1 - \alpha)\mathcal{N}(s^*)$) and to invest in safe assets if risky assets are inferior and $\theta$ is high (with probability $(1 - \alpha)(1 - \mathcal{N}(s^*))$). In this case, expected output is

$$Y_S(\phi) = px + \alpha psyr + (1 - \alpha) [(1 - \mathcal{N}(s^*)+s^*)esy_s + \mathcal{N}(s^*)pryr]$$

It is straightforward to see that, if $\mathcal{N}(s^*) = 0$, then $Y_S(\phi) = px + \alpha psyr + (1 - \alpha)psy_s > Y_D(\phi)$, by Assumption 1.1. In essence, when banks are always disciplined by reputation and invest in superior risky assets when those are available and in safe assets when superior risky assets are not available, their expected output is larger without regulation. In contrast, if $\mathcal{N}(s^*) = 1$, then $Y_S(\phi) = px + \alpha psyr + (1 - \alpha)pryr < Y_D(\phi)$, by Assumption 1.2. In essence, when banks are never disciplined by reputation and invest in superior risky projects when those are available and in inferior risky assets when superior risky assets are not available, their expected output is larger with regulation.

This implies that total output in the economy will critically depend on the financing choices of banks with different levels of reputation $\phi$, and the distribution of reputation in the economy. Defining $Pr(S|\phi)$ the probability a bank with reputation $\phi$ uses shadow banking, total expected output in the economy is,

$$Y = \int_0^1 [Pr(S|\phi)Y_S(\phi) + (1 - Pr(S|\phi))Y_D(\phi)] d\phi$$

In the next section I discuss the financing choices of banks with different reputation level $\phi$ and in Section 4 I simulate the effects of these financing choices in total output.
2.3.4 Expected economic conditions and risk-taking

How does the deterioration of expected economic conditions, captured by a lower \( \mu \), affect banking, risk-taking, total expected production, and the likelihood of default in the economy? In this section, I show that bad news about the future has the potential to induce a flight of investors from shadow banking to traditional banking, reduce output and increase the default rate of securities.

More precisely, a reduction in economic prospects, \( \mu \), triggers two effects. The first effect is mechanical: A lower \( \mu \) reduces the ex-ante probability that banks invest optimally for a given cutoff \( s^\ast(\phi) \). The second effect is strategic: A lower \( \mu \) leads to a higher cutoff \( s^\ast(\phi) \), making banks less willing to invest optimally for a given \( \theta \). Since the first effect is obvious, the next proposition focuses on the second effect.

**Proposition 2** The cutoff \( s^\ast(\phi) \) decreases monotonically with \( \mu \).

**Proof** The proof applies for any \( \phi \), hence for notational simplicity I denote \( s^\ast(\phi) \) just as \( s^\ast \). Differentiating the condition (6) that pins down \( s^\ast \) with respect to \( \mu \),

\[
\frac{\partial E_\theta|s^\ast \left[V(\phi', \theta|\tilde{\tau}(s^\ast))\right]}{\partial s^\ast} ds^\ast \frac{ds^\ast}{d\mu} = \frac{\partial R(\phi|s^\ast)}{\partial s^\ast} ds^\ast \frac{ds^\ast}{d\mu} + \frac{\partial R(\phi|s^\ast)}{\partial \mu}.
\]

Then,

\[
\left(\frac{\partial E_\theta|s^\ast \left[V(\phi', \theta|\tilde{\tau}(s^\ast))\right]}{\partial s^\ast} - \frac{\partial R(\phi|s^\ast)}{\partial s^\ast}\right) ds^\ast \frac{ds^\ast}{d\mu} = \frac{\partial R(\phi|s^\ast)}{\partial \mu}. \tag{8}
\]

By assumptions 2 and 3, the term in parentheses is positive. Since \( \frac{\partial N(s^\ast)}{\partial \mu} < 0 \), then \( \frac{\partial R(\phi|s^\ast)}{\partial \mu} < 0 \), and the left hand side is negative, which implies that \( \frac{ds^\ast}{d\mu} < 0 \). Q.E.D.

Intuitively, a decline in \( \mu \) increases \( R(\phi|s^\ast) \) for a given \( s^\ast \) (by an increase in the cumulative distribution up to \( s^\ast \)). This requires a larger \( s^\ast \) to raise \( E_\theta|s^\ast \left[V(\phi', \theta|\tilde{\tau}(s^\ast))\right] \) and fulfill equation (6). This direct effect increases \( s^\ast \). Furthermore, an increase in \( s^\ast \) implies a further increase in \( R(\phi|s^\ast) \), which reinforces the direct effect generated by a lower \( \mu \). There is also a second effect that comes from reducing beliefs \( \tilde{\tau}(s_i) \) and reputation updating at each \( s_i \), (since \( \tilde{\tau}(s_i) = 1 - \Phi(\sqrt{s_i - \mu}) \), weakly reducing \( E_{\theta|s_i} \left[V(\phi', \theta|\tilde{\tau}(s_i))\right] \), for every signal \( s_i \). Hence, a further increase in \( s^\ast \) is necessary to compensate for this reduction and still fulfill equation (6).
At this point we can distinguish between informative and uninformative news about changes in expected economic conditions. In this model news about future fundamentals induce changes in real activity by affecting credit markets, including cases in which news are pure noise and do not reflect real fundamental changes. Figure 2 shows the effects of uninformative bad news (lower expected fundamental $\mu$ without a real change in the distribution of fundamentals). This wave of pessimism induces less production by changing banks’ strategic risk-taking behavior and driving some activity to traditional banking. In contrast, Figure 3 shows the effects of informative bad news (a real reduction of $\mu$), which also decreases output, both mechanically (larger ex-ante probability of risk-taking) and strategically, driving even more activity to traditional banking.

2.4 Financing through Traditional or Shadow Banking?

Now I study under which expected fundamentals, $\mu$, banks with reputation $\phi$ choose to use traditional banking or shadow banking. When reputation incentives are strong (this is, when $\mu$ is large), lenders’ confidence about self-regulation prevails and securitization provides a feasible way to avoid restrictive regulation because investors are willing to participate at low rates. The next proposition, proved in the Appendix, summarizes the result.

**Proposition 3** Optimal Financing Decisions

Assume $\gamma_s \to \infty$. If $\mu \in \mathbb{R}$, there is a unique cutoff $\mu^*(\phi)$ such that banks with reputation $\phi$ issue securities when $\mu \geq \mu^*(\phi)$ and issue debt when $\mu < \mu^*(\phi)$. If the following condition is
satisfied, then shadow banking is never used (this is, $\mu^*(\phi) = \infty$).

\begin{equation}
    p_s\alpha(y_r - y_s) + \beta\hat{p}_DE_{\theta|\mu=\infty}[V(\phi', \theta) - V(\phi, \theta)] \geq \frac{(1 - \phi)(1 - p_x)(1 - \alpha)(p_s - p_r)}{\hat{p}_B + \phi(1 - p_x)(1 - \alpha)(p_s - p_r)} \tag{9}
\end{equation}

An important corollary of the previous proposition, when the range of possible $\mu$ is restricted is the following:

**Corollary 1** Assume $\mu \in [\underline{\mu}, \overline{\mu}]$. If $\mu > \mu^*(\phi)$, banks with reputation $\phi$ always issue securities and raise funds with shadow banking. If $\mu < \mu^*(\phi)$, banks with reputation $\phi$ always issue debt and raise funds with traditional banking.

Figure 4 illustrates the main properties of expected profits when banks issue debt and when they issue securities. I also show the threshold $\mu^*(\phi)$ that defines the regions under which debt and securities are preferred. Intuitively, when good banks are optimistic about future conditions (this is, $\mu > \mu^*(\phi)$), they value reputation and invest safely enough to credibly guarantee securities as if they were regulated. This system, however, has a chance of collapse in case fundamentals reveal to be weaker than expected, since good banks would rather take excessive risks.

At the other extreme, when good banks are pessimistic about future conditions (this is, $\mu < \mu^*(\phi)$), they do not value reputation and do not have incentives to invest optimally. Lenders are aware of this lack of banks' incentives and understand that the quality of banks' assets that sustain guarantees is not as if banks were regulated. Then lenders require higher rates for their funds in compensation for taking higher risks in shadow banking, which make banks better off by raising funds through traditional banking.

The previous analysis characterizes the decisions of good banks, but bad banks always pool with good banks and take the same financing choices. If good banks raise funds with securities, bad banks also issue securities, which give them the possibility of investing only in risky assets at an interest rate subsidized by the presence of good banks. If good banks raise funds with debt, bad banks also issue debt, otherwise their reputation gets lost immediately and either they have to finance with debt anyways, or if financing with securities they have to borrow facing the highest possible interest rate $\frac{1}{\hat{p}_B}$, receiving the lowest possible reputation $\phi' = 0$ forever in the future.
In the next proposition, I consider how $\mu^*(\phi)$ varies for different reputation levels $\phi \in [0, 1]$. This comparison is important to understand how the fragility of the financial system is endogenous to the distribution of reputation levels in the economy. The proof is in the Appendix.

**Proposition 4** Thresholds $\mu^*(\phi)$ decrease with reputation $\phi$.

Intuitively, for a given expectation of future conditions $\mu$, banks with good reputations have larger reputation concerns, and their guarantees to cover securities in distress with high quality assets are more credible to lenders. Then lenders charge a lower rate for securities and high reputation banks are better off raising funds with shadow banking and avoid restrictive regulations. The next proposition, which arises from combining Propositions 3 and 4 for a given $\mu$, summarizes this result.

**Proposition 5** Given expected future economic conditions, $\mu$, there is a $\phi^*(\mu)$ such that all
banks with reputation $\phi > \phi^*(\mu)$ issue securities and participate in shadow banking while all banks with reputation $\phi < \phi^*(\mu)$ issue debt and participate in traditional banking.

3 Regulation that Enhances Reputation Forces

Capital requirements are beneficial because they provide firewalls against excessive risk-taking, but are costly because they can choke off potential good investment opportunities. Securitization is an option to avoid restrictive regulation, spawning a shadow banking sector that is preferred but also fragile and volatile.

I now propose novel regulatory tools, that I call novel regulation, which complement the use of capital requirements, to which I refer as standard regulation. Specifically, I argue that a budget balanced scheme of taxes and subsidies that cross-subsidize banks with different reputation levels can enhance the disciplining effects of reputation concerns among those banks that self-select into shadow banking, increasing expected production, and at the same time, making the financial system less sensitive to news about future economic conditions.

Assume the economy does not have aggregate shocks (there is a unique possible $\mu$). I relax this assumption when illustrating my results using simulations in the next section. First, assume that the government can impose taxes and subsidies conditional on $\theta$ for each $\phi$, $T(\phi|\theta)$, such that $\tilde{V}(\phi) = V(\phi, \theta)T(\phi|\theta)$ is fixed. In this case, the results so far can be recomputed considering $\tilde{V}(\phi)$, independent of $\theta$. This potential taxation is compelling but difficult to sustain. Financial decisions and crises are intimately related to news about expected economic conditions, so it is usually not plausible to eliminate financial cycles by eliminating economic cycles and news about the future directly. This raises a more challenging question: Is it possible to reduce fragility if it is not possible to attack the source of fragility directly?

Another, ideal but unfeasible solution, is to just give a high subsidy to all banks, regardless of their reputation $\phi$, conditional on their repayment of the loans, such that $\tilde{V}(\phi) = V(\phi, \theta)T$ with $T > 1$ for all $\phi$ and $\theta$. This naturally increases the cost of default for all banks and then allows for more self-regulation. This solution has the same effects as an exogenous increase of $\mu$, but how does one finance these widely available subsidies?
In order to at least partially finance the proposed policy, a government needs to transfer resources across reputation levels, subsidizing banks of relatively high reputation (this is $T(\phi) > 1$ for $\phi > \bar{\phi}$) and taxing banks of relatively low reputation (this is $T(\phi) < 1$ for $\phi < \bar{\phi}$). I assume, then, a subsidy scheme independent on $\theta$ and monotonically increasing in reputation, with a level of reputation $\bar{\phi}$ for which $T(\bar{\phi}) = 1$ and not tax or subsidy is applied.

This novel regulation increases the incentives for banks with low reputation to issue debt and use regulated traditional banking, and increase the incentives for banks with high reputation to use shadow banking. Furthermore, the incentives for banks participating in shadow banking to invest optimally are larger and less sensitive to news about future economic prospects, inducing a more stable financial system and overall higher production. The next proposition, proved in the Appendix, summarizes this result.

**Proposition 6** Define $\phi^*(\mu)$ as the reputation level that makes good banks indifferent between shadow and traditional banking in the absence of cross subsidization and $\phi^{**}(\mu)$ the reputation level that makes good banks indifferent in the presence of cross subsidization. There is a subsidy scheme increasing in reputation, $\frac{\partial T(\phi)}{\partial \phi} > 0$ such that $T(\widehat{\phi}) = 1$, where $\phi^*(\mu) < \widehat{\phi} < \phi^{**}(\mu)$ and $\phi^*(\mu) = \phi^{**}(\mu)$.

Furthermore, the expected gains from assets invested by banks with reputation $\phi < \phi^*(\mu)$ remain unchanged, while expected gains from assets invested by banks with reputation $\phi > \phi^*(\mu)$ increase, since their ex-ante probability of excessive risk-taking ($N(s^*(\phi, \mu))$) decline.

Intuitively, a cross-subsidization scheme just affects behavior of those banks that use shadow banking in equilibrium (those with relatively high reputation), and not banks that use traditional banking in equilibrium, and are then regulated. Hence, this novel intervention hinges on subsidizing shadow banking and making it sustainable using funds from banks that already use a stable, standard, regulated banking system. In a sense, good banks cannot effectively enjoy their capacity to self-regulate because lenders confuse them with bad banks. Since bad banks on average have lower reputation levels than good banks, taxing low reputation is a way for bad banks to compensate the externality they impose over good banks.

Naturally, whether the cross-subsidization policy can be self-financed or not depends on the distribution of reputation across banks and on the value functions for different
reputation levels. In particular, cross-subsidization is sustainable without external funds for the government at each expected future condition $\theta$ if

$$\int_0^1 T(\phi)V(\phi, \theta)d\phi = \int_0^1 V(\phi, \theta)d\phi.$$  

where $d\phi$ is the distribution of reputation, conditional on a realized fundamental $\theta$.

**Caveats:** This proposed novel regulation is just intended to highlight important forces that go towards making shadow banking more sustainable. It is clear, however, that it presents serious challenges in terms of implementation and political feasibility. If bank size is correlated with reputation, for example, subsidizing large banks with taxes to small banks may be in direct conflict with other concerns, such as avoiding market power and concentration in the banking industry.

Given these implementation concerns, it is interesting to note that bailouts may be a hidden type of cross-insurance that indeed stabilizes the banking system. Most literature claims that bailouts buffer the losses of “too big to fail” banks, then inducing their excessive risk-taking. This literature, however, takes the size of banks as exogenous. If becoming a big bank requires building and maintaining good reputations, bailouts may have the opposite effect of increasing the benefits of size and the incentives to become big and to remain big, then reducing the incentives for excessive risk-taking of “too big to fail” banks.

Finally, even though this alternative novel regulation improves welfare, there may exist other more complicated capital requirement regulations (for example that fluctuate cyclically and only bind when reputation concerns weaken) that constitute even better policies. Regardless of its practicality and optimality, the novel regulation proposed here uncovers potential benefits of cross-subsidization in the banking industry, which compensates other considerations raised in the literature. I leave for future research the practical complications of implementing a cross-subsidization policy and the design of an optimal mechanism that maximizes welfare by acknowledging the role of shadow banking.
4 Simulations

In this section I illustrate the results from the model using a numerical example. I assume that continuation values are linearly increasing in $\phi$ and $\theta$. In this case, even when payoffs are linear, reputation formation convexifies the schedule of cutoffs that determine the financing choices, inducing sudden and dramatic increases in risk-taking and collapses in shadow banking, even without obvious declines in expected economic conditions. Furthermore, shadow banking can collapse even in the absence of real fundamental changes; just in response to small and uninformative bad news.

I assume the following parameters: The probability of success is $p_s = 0.5$ for safe and superior risky assets, $p_r = 0.1$ for inferior risky assets, and $p_x = 0.2$ for new assets. Payoffs in case of success are $y_s = 3$ for safe assets, $y_r = 10$ for risky assets and $x = 15$ for new assets. A risky asset is superior with probability $\alpha = 0.1$, discounting is $\beta = 0.99$, the variance of fundamentals is $\sigma^2 = 2$, signals about fundamentals are very precise $\gamma_s \rightarrow \infty$, and $V(\phi, \theta) = k \phi \theta$ with $k = 0.5$. These parameters fulfill Assumption 1. Finally, I assume a uniform distribution of reputation levels in the market.

4.1 Static Results

Figure 5 shows the expected profits from financing using debt and securitization for a good bank with reputation $\phi = 0.5$. The bank chooses to finance with securities and avoid regulation for all $\mu$ greater than $\mu^* = 3.65$. Figure 6 shows the interest rates for debt and securities. Rates for debt, $R_D$, do not depend on expected fundamentals or future prospects because the probability of default is independent of $\mu$. In contrast, rates for securities $R_S$ critically depend on expected future prospects, suddenly increasing from 2.2 to 3.2 as $\mu$ declines from 6 to 1. Figure 7 illustrates the ex-ante probability that good banks take excessive risks when raising funds with shadow banking, highlighting the source of interaction between the rate of securities $R_S$ and expected future prospects $\mu$ highlighted in Figure 6.

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18 There is nothing particular about the functional form of $V$, as long as it is increasing in $\phi$ and $\theta$.
19 This is a conservative assumption since bad banks default and exit more often, which means the distribution of reputation is skewed towards higher reputation levels, hence creating even more clustering in financing and risk-taking decisions than I illustrate here.
Finally, while Figure 5 shows thresholds $\mu^*(\phi)$ only for banks with reputation $\phi = 0.5$, Figure 8 shows those thresholds for all reputation levels. The schedule of thresholds is convex in expected fundamentals, even though continuation values are linear in fundamentals. The reason, as is clear from Figure 1, is that learning is stronger for intermediate reputation levels and weaker for extreme reputation levels.²⁰

4.2 Dynamic Results

Now I simulate 100 periods of this economy to illustrate the dynamics of shadow banking, its impact on total production and default and its potential collapse. An important dynamic in the economy is how agents learn about expected economic con-

²⁰This is a more general property once continuation values are determined endogenously, as shown in Ordonez (2013b).
ditions. The previous static results show that poor current economic conditions lead to more expectations of risk taking, less securitization, and more constrained shadow banking. In this section I show that poor current economic conditions also lead agents to infer conditions will be poor in the future as well. This implies that times with high default are followed by a collapse in securitization and shadow banking. I also illustrate the effects of the novel regulation on production and default.

Assume highly persistent normal times and uncommon recessions. These states are characterized by two possible distributions, with means $\mu_N = 3$ and $\mu_R = -1$, which follow a Markov process with persistence parameters $\lambda_N = 0.98$ and $\lambda_R = 0.70$.

Agents observe the realized $\theta_t$ at the end of each period and uses it to estimate the current distribution ($\mu_t$) and the expected distribution next period ($E(\mu_{t+1})$). The probability that the economy faces a recession, conditional on $\theta_t$, is

$$Pr(\mu_t = \mu_R|\theta_t) = \frac{f(\theta_t|\mu_R)\hat{\mu}_t}{f(\theta_t|\mu_R)\hat{\mu}_t + f(\theta_t|\mu_N)(1 - \hat{\mu}_t)},$$

where $f(\theta_t|\mu_R)$ is the density of $\theta_t$ in recessions, when the mean is $\mu_R$ and the precision is $\gamma_{\theta}$, and $\hat{\mu}_t$ is the prior probability that $\mu_t = \mu_R$. Finally, the posterior $\hat{\mu}'_t$ is used to estimate the probability that $\mu_{t+1} = \mu_R$ from the Markov process,

$$Pr(\mu_{t+1} = \mu_R|\hat{\mu}'_t) = \lambda_R\hat{\mu}'_t + (1 - \lambda_R)(1 - \hat{\mu}'_t).$$

This probability is used to estimate the expected distribution that generates the fundamental $\theta$ next period

$$\hat{\mu}_{t+1}N(\mu_R, \frac{1}{\gamma_{\theta}}) + (1 - \hat{\mu}_{t+1})N(\mu_N, \frac{1}{\gamma_{\theta}}) = N(\bar{\mu}_{t+1}, \sigma_{t+1}),$$

where $\bar{\mu}_{t+1} = \hat{\mu}_{t+1}\mu_R + (1 - \hat{\mu}_{t+1})\mu_N$ and $\sigma_{t+1} = \left[\hat{\mu}_{t+1}^2 + (1 - \hat{\mu}_{t+1})^2\right]\frac{1}{\gamma_{\theta}}$.

Note that, for simplicity, I constructed the exercise such that the updating of economic conditions ($\hat{\mu}'$) and the updating of reputation ($\phi'$) are independent of each other, and the only link across periods is the learning about economic conditions. Figure 9 shows 100 simulated fundamentals $\theta_t$ from simulated distributions of fundamentals $\mu_t$, and expected distribution of fundamentals $\bar{\mu}_t$ at each period $t$, following the previously described learning process about economic conditions.
In the next figures I compare thresholds, the scope of shadow banking, and total output with and without the novel regulation. I assume a subsidy schedule of the form $T(\phi) = a \phi^b$, such that $\widehat{V}(\phi', \theta) = a \phi^b V(\phi', \theta)$. Given the linearity of value functions and the uniform distribution of reputation, this scheme is budget balanced if

$$\int_0^1 a k \theta \phi^{b+1} d\phi = \int_0^1 k \theta \phi d\phi,$$

which happens when $a = 1 + \frac{b}{2}$. Specifically, in what follows I compare the situation without the novel regulation ($b = 0$) with a case of novel regulation in which $b = 2$, and then $a = 2$. In this case $T(\phi) = 2 \phi^2$ and then $T(\phi) = 1$ for $\phi = 0.71$.

Figure 10 shows the expected profits of a bank with reputation $\phi = 0.5$ with and without taxation to implement the novel regulation I propose. The functions without novel regulation are the same as in Figure 5, and depicted lightly. In contrast to the absence of cross subsidization, in which banks with reputation $\phi = 0.5$ securitize if $\mu > 3.65$, when there is cross subsidization banks with reputation $\phi = 0.5$ securitize if $\mu > 5$. Intuitively, as discussed in the previous section, the novel regulation increases expected future values for all reputation levels above $\phi = 0.71$. This implies that all good banks with reputation $\phi < 0.71$ have less incentives to invest optimally if raising
funds in the shadow banking.

Figure 11 shows the thresholds for all reputation levels with and without the novel regulation. High reputation levels are more likely to securitize with cross-subsidization, while lower reputation levels are more likely to raise debt and be subject to standard regulation. Since the schedule of thresholds is less sensitive to changes in $\mu$, the economy is less subject to shocks to future economic conditions.

Note that I choose the subsidy scheme to maintain $\phi^*(\mu = 3) = \phi^{**}(\mu = 3) = 0.6$. To see why, take a bank with current reputation $\phi = 0.6$. If reputation is not updated then $\phi' = 0.6$ and $T(\phi' = 0.6) = 2(0.6)^2 = 0.72$, which implies the bank has to pay a fraction 28% of its value function at the end of the period. In the case reputation is updated because banks invest optimally and repay, then $\phi' = 0.76$ and $T(\phi' = 0.76) = 2(0.76)^2 = 1.16$, which implies the bank receives a subsidy of 16% of its value function at the end of the period. These two effects exactly compensate each other such that banks with reputation $\phi = 0.6$ are still indifferent between participating in traditional and shadow banking.

Figure 12 shows the fraction of banks in the economy that securitize every period. Most of the time, since $\mu = 3$ the level of securitization is the same with or without the novel regulation. As is clear in Figure 11, when $\mu = 3$ both with and without novel regulation, all banks with reputation above $\phi = 0.6$ securitize. Given the uniform distribution of reputations this implies that 40% of banks choose to finance through shadow banking.

When there is novel regulation, however, there is a smaller decline in shadow banking.
when the expected $\mu$ declines. In essence, with cross subsidization, high reputation banks have more incentives to invest optimally, which sustains shadow banking in the presence of adverse shocks to expected future conditions.

Figure 13 shows expected production of old assets in the economy with and without the novel regulation. With cross-subsidization there are more incentives to invest optimally for banks with high reputation, which are those who select into shadow banking. Since expected profits for the banks that do not securitize are subject to standard regulation and generate the same expected profits, total production does not suffer as it does without the novel regulation. This implies that the right combination between the novel and the standard regulation has the potential to increase expected production in all states of the world.

Interestingly, the only time the economy enters into a recession ($\mu_R$) is during periods 80-82. However, there are negative shocks to production also in periods 45 and 61, not because of a recession but just because bad news about the economy. These irrelevant shocks make individuals revise the probability of a recession upward, reducing reputation concerns and inefficiently discouraging shadow banking.

To put these results in context, it is important to highlight that the expected production from old assets when all banks invest optimally is 1.85, production when all banks take excessive risk is 1.4 and production when all banks raise debt and then are subject to standard regulation and capital requirements that make them invest only in safe assets is 1.5. This shows the benefits of standard regulation in the absence of reputation concerns, which is raising the old assets production from 1.4 to 1.5. This
also shows the benefits and fragility of shadow banking in the presence of reputation concerns, raising production from 1.5 to 1.58 in normal times, but causing sudden reductions under bad news. Finally, this exercise also shows the benefits of the novel regulation in increasing old assets production from 1.58 to 1.585 in normal times and buffering the losses in case of bad news.

5 Conclusions

This paper provides a new view of shadow banking, not as an inherently dangerous system without regulatory firewalls, but as a positive signal that self-regulation exists in financial markets and that agents can get rid of inefficient government restrictions. Still, since self-regulation is provided by the market through reputation concerns, it is fragile because the value of reputation depends on news about future economic prospects and may collapse even with seemingly irrelevant news. Hence, the gains of shadow banking in terms of efficiency should be evaluated against its fragility.

The natural question is how to enhance the benefits of shadow banking and at the same time reduce its costs. I show that banks with low reputation and lack of discipline self-select into the use of traditional banking and those with high reputations self-select into shadow banking. I propose a novel regulation that combines traditional capital requirements with cross-subsidizations and explore this policy. With the novel regulation, banks that do not use the shadow banking are still restrained by regulation while banks that do use the shadow banking are less sensitive to news about economic conditions, making shadow banking more stable.

The insight that shadow banking arises endogenously when self-regulation becomes feasible is also critical to evaluate the frequent proposals for more stringent government regulations. Indeed, trying to extend regulations to more banking activities can backfire into the undesirable creation of “new shadow banking activities” with banks less concerned for their reputation and more fragile in their risk-taking behavior.

With the goal of exploring how reputation and regulation interact to boost and shrink shadow banking by changing banking behavior, this paper unfortunately leaves aside other interesting aspects of financial crises. In this paper, for example, the transitions between traditional and shadow banking happen in a relatively orderly way, not capturing the dynamics of crises driven by bubbles or runs. Similarly, the paper is not
intended as a model of securitization and its details, but rather as a model about the use of securitization for regulation arbitrage and its effects. Combining these elements into a single setting can be a fruitful endeavor for future research.

The model can be extended in several directions as well: First, reputation gains could be obtained endogenously, as Ordonez (2013b) implemented in a different setting; second, the same forces could be accommodated to study other financial institutions and instruments, such as repo, money markets, investment banks, etc; third, the model could be extended to study transactions that involve counter party risks and collateral of unknown quality, where prices also depend on aggregate conditions.

All these extensions would make the model richer and more realistic, but would not change the main insight: reputation introduces discipline that restricts risk more efficiently than government regulations. In this sense, reputation concerns allow for the rise of shadow banking as a superior, but more fragile, alternative to traditional banking. Whether it is desirable to have a system based on self-regulation depends on the trade-off between output and volatility. Hence, the challenge for regulation is not to eliminate shadow banking, but to make it sustainable and more stable.

References


A Appendix

A.1 Proof of Proposition 3

Value functions from issuing debt and securities depend on $\phi$ and $\mu$. If there is almost perfect information ($\gamma_s \to \infty$), these value functions are arbitrarily closely approximated by

$$U_D^G(\phi, \mu) = p_x x + p_s y_s + \hat{p}_D [\beta E_{\theta|\mu} V(\phi, \theta) - R_D]$$

and

$$U_S^G(\phi, \mu) = p_x x + \alpha p_s y_r + (1 - \alpha) [N(s^*(\phi, \mu)) p_r y_r + (1 - N(s^*(\phi, \mu))) p_s y_s]$$

$$+ N(s^*(\phi, \mu)) \hat{p}_G(\hat{\tau} = 0) [\beta E_{\theta|\mu, \theta < s^*} V(\phi, \theta) - R_S(\phi|s^*(\phi, \mu))]$$

$$+(1 - N(s^*(\phi, \mu))) \hat{p}_G(\hat{\tau} = 1) [\beta E_{\theta|\mu, \theta > s^*} V(\phi', \theta) - R_S(\phi|s^*(\phi, \mu))]$$

respectively, where $\hat{p}_G(\hat{\tau} = 0) = \hat{p}_B$, $\hat{p}_G(\hat{\tau} = 1) = \hat{p}_D$ and $\phi'$ is the updated reputation computed with $\hat{\tau} = 1$.

When $\mu$ is sufficiently low, such that $N(s^*(\phi, \mu)) \to 1$, then it is always optimal for good banks to issue debt and be subject to regulation (this is, $U_D^G(\phi, \mu) > U_S^G(\phi, \mu)$ for all $\phi$). This result is straightforward from replacing $N(s^*(\phi, \mu)) \to 1$ into equation (11), since by assumption $p_s y_s > \alpha p_s y_r + (1 - \alpha)p_r y_r$ and $\hat{p}_D > \hat{p}_B$. This implies there is always a range of $\mu$ low enough such that banks take excessive risk almost with certainty (this is $N(s^*(\phi, \mu)) \to 1$), and banks participate in traditional banking.

When $\mu$ is sufficiently high such that $N(s^*(\phi, \mu)) \to 0$ then it is optimal for good banks to issue securities and avoid regulation only if $U_D^G(\phi, \mu) < U_S^G(\phi, \mu)$. From replacing $N(s^*(\phi, \mu)) \to 0$ into equation (11), this happens only under condition (9) evaluated at $\mu$. This implies that, if $\mu$ is such that good banks invest optimally almost with certainty (this is $N(s^*(\phi, \bar{\mu})) \to 0$), then banks raise securities only if condition (9) evaluated at $\mu$ is fulfilled. If not, then banks raise debt.

Assuming condition (9) for some range of $\mu$, there are values of $\mu$ low enough such that $U_D^G(\phi, \mu) > U_S^G(\phi, \mu)$ and values of $\mu$ high enough such that $U_D^G(\phi, \mu) < U_S^G(\phi, \mu)$. Now, I show that there is a unique threshold $\mu^*$ at which $U_D^G(\phi, \mu^*) = U_S^G(\phi, \mu^*)$.

Taking derivatives of the banks’ expected profits with respect to $\mu$,

$$\frac{\partial U_D^G(\phi, \mu)}{\partial \mu} = \beta \hat{p}_D \frac{\partial E_{\theta|\mu} V(\phi, \theta)}{\partial \mu} > 0$$

(12)
while
\[
\frac{\partial U_G^S(\phi, \mu)}{\partial \mu} = N(s^*(\phi, \mu))\beta \hat{p}_B \frac{\partial E_{\theta|\mu, \theta<s^*}V(\phi, \theta)}{\partial \mu} + (1 - N(s^*(\phi, \mu)))\beta \hat{p}_D \frac{\partial E_{\theta|\mu, \theta>s^*}V(\phi', \theta)}{\partial \mu}
\]
\[
+ \frac{\partial N(s^*(\phi, \mu))}{\partial \mu} [(1 - \alpha)(p_r y_r - p_s y_s)]
\]
\[
+ \frac{\partial N(s^*(\phi, \mu))}{\partial \mu} \beta \hat{p}_B [E_{\theta|\mu, \theta<s^*}V(\phi, \theta) - R_S(\phi)s^*(\phi, \mu)]
\]
\[
- \frac{\partial N(s^*(\phi, \mu))}{\partial \mu} \beta \hat{p}_D [E_{\theta|\mu, \theta>s^*}V(\phi', \theta) - R_S(\phi)s^*(\phi, \mu)]
\]
\[
- [N(s^*(\phi, \mu))\beta \hat{p}_B + (1 - N(s^*(\phi, \mu)))\beta \hat{p}_D] \frac{\partial R_S(\phi)s^*(\phi, \mu)}{\partial \mu} \frac{\partial N(s^*(\phi, \mu))}{\partial \mu}
\]
\]

Adding and subtracting the following expression
\[
\beta \hat{p}_D \frac{\partial E_{\theta|\mu}V(\phi, \theta)}{\partial \mu} \equiv \beta \hat{p}_D \left[ N(s^*) \frac{\partial E_{\theta|\mu, \theta<s^*}V(\phi, \theta)}{\partial \mu} + (1 - N(s^*)) \frac{\partial E_{\theta|\mu, \theta>s^*}V(\phi, \theta)}{\partial \mu} \right]
\]
to the first term of equation (13), then that first term can be rewritten as
\[
\beta \hat{p}_D \frac{\partial E_{\theta|\mu}V(\phi, \theta)}{\partial \mu} - \beta N(s^*(\phi, \mu))(\hat{p}_D - \hat{p}_B) \frac{\partial E_{\theta|\mu, \theta<s^*}V(\phi, \theta)}{\partial \mu}
\]
\[
+ \beta(1 - N(s^*(\phi, \mu)))\hat{p}_D \frac{\partial E_{\theta|\mu, \theta>s^*}[V(\phi', \theta) - V(\phi, \theta)]}{\partial \mu} > 0
\]

From Proposition 2, \( \frac{\partial N(s^*(\phi, \mu))}{\partial \mu} < 0 \). Then, the second term and the sum of the third and fourth terms of equation (13) are positive too. Finally, the last term is also positive because \( \frac{\partial R_S(\phi)s^*(\phi, \mu)}{\partial N(s^*(\phi, \mu))} > 0 \). Hence, \( \frac{\partial U_G^S(\phi, \mu)}{\partial \mu} > 0 \). Furthermore, when \( N(s^*(\phi, \mu)) \to 1 \),
\[
\frac{\partial U_G^S(\phi, \mu)}{\partial \mu} \approx \beta \hat{p}_B \frac{\partial E_{\theta|\mu}V(\phi, \theta)}{\partial \mu} < \frac{\partial U_G^D(\phi, \mu)}{\partial \mu}
\]
and when \( N(s^*(\phi, \mu)) \to 0 \),
\[
\frac{\partial U_G^S(\phi, \mu)}{\partial \mu} \approx \beta \hat{p}_D \frac{\partial E_{\theta|\mu}V(\phi', \theta)}{\partial \mu} > \frac{\partial U_G^D(\phi, \mu)}{\partial \mu}
\]

In words, expected profits from raising funds, both in traditional and shadow banking, increase with \( \mu \). Furthermore, when \( \mu \) is relatively low, expected profits with securities increase at a lower rate than expected profits with debt. In contrast, when \( \mu \) are relatively high, expected profits with securities increase at a faster rate than expected profits with debt.
Given the symmetry of the normal distribution, since \( \frac{\partial N(s^*(\phi, \mu))}{\partial \mu} < 0 \) from Proposition 2 and the fact that the second, third, and fourth terms of equation (13) are positive, it is straightforward to see that \( \frac{\partial U^S_\mu(\phi, \mu)}{\partial \mu} \) is increasing for \( N(s^*(\phi, \mu)) > 0.5 \) and that \( \frac{\partial U^S_\mu(\phi, \mu)}{\partial \mu} \) is decreasing for \( N(s^*(\phi, \mu)) < 0.5 \) (normal densities are decreasing for \( N(s^*(\phi, \mu)) > 0.5 \) and increasing for \( N(s^*(\phi, \mu)) < 0.5 \)). This last statement implies that the lower bound for \( \frac{\partial U^S_\mu(\phi, \mu)}{\partial \mu} \) when \( N(s^*(\phi, \mu)) < 0.5 \) is the expression (15). Then, if condition (9) holds, the expected profits with securities cross expected profits with debt from below only once, and there is only a cutoff \( \mu^*(\phi) \) for which \( U^S_G(\phi, \mu^*) = U^D_G(\phi, \mu^*) \).

### A.2 Proof of Proposition 4

Taking derivatives of banks’ expected profits with respect to \( \phi \),

\[
\frac{\partial U^D_G(\phi, \mu)}{\partial \phi} = \beta \hat{\phi}_B \frac{\partial E_{\theta|\mu} V(\phi, \theta)}{\partial \phi},
\]

which is positive by construction of value functions increasing with \( \phi \).

\[
\frac{\partial U^S_G(\phi, \mu)}{\partial \phi} = N(s^*(\phi, \mu)) \beta \hat{\phi}_B \frac{\partial E_{\theta|\mu, \theta<s^*} V(\phi, \theta)}{\partial \phi} + (1 - N(s^*(\phi, \mu))) \beta \hat{\phi}_D \frac{\partial E_{\theta|\mu, \theta>s^*} V(\phi, \theta)}{\partial \phi} \frac{\partial \phi'}{\partial \phi} \\
+ N(s^*(\phi, \mu)) \beta \hat{\phi}_B \frac{\partial E_{\theta|\mu, \theta<s^*} V(\phi, \theta)}{\partial s^*} \frac{\partial s^*}{\partial \phi} + (1 - N(s^*(\phi, \mu))) \beta \hat{\phi}_D \frac{\partial E_{\theta|\mu, \theta>s^*} V(\phi, \theta)}{\partial s^*} \frac{\partial s^*}{\partial \phi} \\
+ \frac{\partial N(s^*(\phi, \mu))}{\partial s^*} \beta \hat{\phi}_B \frac{\partial s^*}{\partial \phi} (1 - \alpha)(p_r y_r - p_s y_s) \\
+ \frac{\partial N(s^*(\phi, \mu))}{\partial s^*} \beta \hat{\phi}_D \frac{\partial s^*}{\partial \phi} (E_{\theta|\mu, \theta<s^*} V(\phi, \theta) - R_S(\phi, \mu)) \\
- \frac{\partial N(s^*(\phi, \mu))}{\partial s^*} \beta \hat{\phi}_D \frac{\partial s^*}{\partial \phi} (E_{\theta|\mu, \theta>s^*} V(\phi, \theta) - R_S(\phi, s^*(\phi, \mu))) \\
- [N(s^*(\phi, \mu)) \beta \hat{\phi}_B + (1 - N(s^*(\phi, \mu))) \beta \hat{\phi}_D] \frac{\partial R_S(\phi, s^*(\phi, \mu))}{\partial s^*} \frac{\partial N(s^*(\phi, \mu))}{\partial \phi} \frac{\partial s^*}{\partial \phi}.
\]

Differentiating the condition that pins down \( s^* \) with respect to \( \phi \),

\[
\frac{d \theta_{\gamma|s^*} [V(\phi', \theta|\tau(s^*))]}{d \phi} = \frac{d[\Psi + R_S(\phi|s^*)]}{d \phi}.
\]

Developing the total derivative

\[
\frac{\partial \beta E_{\theta|s^*} [V(\phi', \theta|\tau(s^*))]}{\partial \phi} \frac{d s^*}{d \phi} + \frac{\partial \beta E_{\theta|s^*} [V(\phi', \theta|\tau(s^*))]}{\partial \phi'} \frac{d \phi'}{d \phi} = \frac{\partial R(\phi|s^*)}{\partial s^*} \frac{d s^*}{d \phi} + \frac{\partial R_S(\phi, s^*)}{\partial \phi}.
\]

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\[
\left[ \frac{\partial \beta E_{\theta \mid s^*} [V(\phi', \theta \mid \hat{\tau}(s^*))]}{\partial s^*} - \frac{\partial R(\phi \mid s^*)}{\partial s^*} \right] \frac{ds^*}{d\phi} = \frac{\partial R_S(\phi \mid s^*)}{\partial \phi} - \frac{\partial \beta E_{\theta \mid s^*} [V(\phi', \theta \mid \hat{\tau}(s^*))]}{\partial \phi} \frac{\partial \phi'}{\partial \phi}.
\]

By assumptions 2 and 3, the term in brackets is positive. In contrast, the right hand side is negative because

\[
\frac{\partial R_S(\phi \mid s^*)}{\partial \phi} = -\frac{(1 - p_e)(1 - \alpha)(p_s - p_r)}{(\hat{p}_B + \phi(1 - \mathcal{N}(s^*))(1 - p_e)(1 - \alpha)(p_s - p_r))^2}(1 - \mathcal{N}(s^*)) < 0,
\]

and \(\frac{\partial \beta E_{\theta \mid s^*} [V(\phi', \theta \mid \hat{\tau}(s^*))]}{\partial \phi'} \frac{\partial \phi'}{\partial \phi} > 0\), again by construction of value functions and Bayesian updating. This implies that \(\frac{ds^*}{d\phi} < 0\).

The first and second terms of equation (17) are positive by construction of the value functions, the sum of the third and fourth terms is positive (since \(\frac{\partial \mathcal{N}(s^* \mid \phi, \mu)}{\partial \phi} > 0\)), and the last term is also positive (since \(\frac{\partial R_S(\phi \mid s^* \mid \phi, \mu)}{\partial \mathcal{N}(s^* \mid \phi, \mu)} > 0\)). This implies that \(\frac{ds^*}{d\phi} > 0\).

Since both \(U^S_G(\phi, \mu)\) and \(U^D_G(\phi, \mu)\) increase with \(\phi\) for each \(\mu\), the threshold \(\mu^*_s(\phi)\) declines with \(\phi\) if \(\frac{\partial U^S_G(\phi, \mu^*_s(\phi))}{\partial \phi} \geq \frac{\partial U^D_G(\phi, \mu^*_s(\phi))}{\partial \phi}\). Since the threshold \(\mu^*_s(\phi)\) is given by the value \(\mu\) at which equations (10) and (11) are equal, by evaluating equations (16) and (17) at \(\mu^*_s\) it is clear that this condition is fulfilled.

### A.3 Proof of Proposition 6

First I prove the impact of subsidies on the incentives to invest optimally in shadow banking, summarized by \(s^*\).

Imposing \(E_{\theta \mid s^*} [V(\phi', \theta \mid \hat{\tau}(s^*))T(\phi')]\) in condition (6) that pins down \(s^*\), and differentiating with respect to \(T(\phi')\),

\[
\frac{\partial \beta E_{\theta \mid s^*} [V(\phi', \theta \mid \hat{\tau}(s^*))T(\phi')]}{\partial T(\phi')} + \frac{\partial \beta E_{\theta \mid s^*} [V(\phi', \theta \mid \hat{\tau}(s^*))T(\phi')]}{\partial s^*} \frac{ds^*}{dT(\phi')} = \frac{\partial R(\phi \mid s^*)}{\partial s^*} \frac{ds^*}{dT(\phi')}.
\]

Then,

\[
\left( \frac{\partial \beta E_{\theta \mid s^*} [V(\phi', \theta \mid \hat{\tau}(s^*))T(\phi')]}{\partial s^*} - \frac{\partial R(\phi \mid s^*)}{\partial s^*} \right) \frac{ds^*}{dT(\phi')} = -\beta E_{\theta \mid s^*} [V(\phi', \theta \mid \hat{\tau}(s^*))].
\]

The right hand side is negative and, by assumptions 2 and 3, the term in brackets is positive, which implies that \(\frac{ds^*}{dT(\phi')} < 0\). In words, the ex-ante probability of risk-taking declines for all reputations \(\phi\) for which the update \(\phi'\) is subsidized with \(T(\phi') > 1\). In contrast, the ex-ante probability of risk-taking increases for all reputations \(\phi\) for which the update \(\phi'\) is taxed with \(T(\phi') < 1\).
This result is important to prove the first part of the proposition. Assume $T(\phi^*) = 1$ such that $T(\phi^{**}) > 1$. This implies that, in the presence of cross-subsidization, good banks with reputation $\phi^*$ strictly prefer to raise funds in the shadow banking. This is clear from comparing equations (10) and (11). Equation (10) remains constant while equation (11) increases for two reasons. First, fixing $s^*$, the value of reputation updating is larger because $E_{\hat{\theta}|\mu,\theta > s^*}V(\phi^{**})T(\phi^{**}) > E_{\hat{\theta}|\mu,\theta > s^*}V(\phi^{**}, \theta)$. Second, as shown above, $s^*$ declines, which reduces $R_S$ and further increases the gains from shadow banking in equation (11).

Combining this result with proposition 4, when $T(\phi^*) = 1$, then $\phi^* > \phi^{**}$. By imposing $T(\phi) < 1$, equation (10) declines while equation (11) does not increase so much as in the previous case. This implies there is always a $\phi^*(\mu) < \phi < \phi^{**}(\mu)$ such that $T(\phi) = 1$ and $\phi^*(\mu) = \phi^{**}(\mu)$. If the government imposes such a scheme, the banks that participate in traditional and shadow banking do not change. The banks with reputation $\phi < \phi^*$ participate in traditional banking and since they are subject to regulation, the keep investing only in safe assets. In contrast, the good banks with reputation $\phi > \phi^*$ participate in shadow banking, but because subsidies decrease $s^*(\phi)$, they invest optimally with a higher probability and shadow banking is less fragile.