Longitudinal Modeling of Insurance Company Expenses

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joint work with
Edward W. (Jed) Frees - University of Wisconsin-Madison

July 31, 2009
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Motivation

- **Expenses by Type**
  - Underwriting expenses: policy acquisition cost, administrative expenses
  - Investment expenses: trading activities, portfolio management
  - Loss adjustment expenses: investigation cost, legal fees

- **Benefits of Expense Analysis**
  - Insurers: rate making, cost control, strategic decision
  - Investors: cost efficiency and profitability analysis
  - Regulators: expense factor, industry benchmark, economic hypothesis

- **Limitations of Current Practice**
  - Ignored three features of insurance company expenses: skewness, negative values and intertemporal dependence
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GOAL:

To develop longitudinal models that can be used for prediction, to identify unusual behavior, and to eventually measure firm inefficiency, by addressing above three features.

Statistical Viewpoint

- Basic regression set-up - almost every analyst is familiar with
  - It is part of the basic actuarial education curriculum
- Incorporating cross-sectional and time patterns is the subject of longitudinal data analysis - a widely available statistical methodology
- Quantile regression focuses on the quantiles of response variable - a relatively new regression technique
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Sampling Procedure

- Firm level data of property-casualty insurers from NAIC
- Observe from 2001 to 2006
- Two types of observations are removed:
  1. Companies with non-positive net premiums written in all years
  2. Records with inactive company status in the last observation year
- Final sample consists of 2,660 companies and 13,925 observations
- Variables in money values are deflated to 2001 US dollars
Distribution of Expenses

Table 1. Descriptive Statistics of Total Expenses ($1,000,000)

<table>
<thead>
<tr>
<th></th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>2,286</td>
<td>2,269</td>
<td>2,303</td>
<td>2,320</td>
<td>2,354</td>
<td>2,393</td>
</tr>
<tr>
<td>Mean</td>
<td>57.01</td>
<td>61.25</td>
<td>64.47</td>
<td>65.37</td>
<td>64.06</td>
<td>63.66</td>
</tr>
<tr>
<td>Median</td>
<td>6.00</td>
<td>6.22</td>
<td>6.01</td>
<td>5.99</td>
<td>5.83</td>
<td>6.12</td>
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<tr>
<td>StdDev</td>
<td>332.03</td>
<td>353.63</td>
<td>364.09</td>
<td>359.50</td>
<td>354.74</td>
<td>352.29</td>
</tr>
<tr>
<td>Minimum</td>
<td>-190.46</td>
<td>-38.16</td>
<td>-32.63</td>
<td>-26.28</td>
<td>-111.08</td>
<td>-20.42</td>
</tr>
<tr>
<td>Maximum</td>
<td>10,410.17</td>
<td>11,307.32</td>
<td>10,966.07</td>
<td>10,397.76</td>
<td>9,809.33</td>
<td>10,051.16</td>
</tr>
</tbody>
</table>

| Percentage of Negative Obs | 5.86% | 6.30% | 6.34% | 5.56% | 6.07% | 5.56% |

- Total expenses are skewed and heavy-tailed distributed
- Lack of balance
- Negative expenses: (1) Adjustment for prior reporting year (2) Reinsurance arrangement
- Strong serial correlation and individual effects
Two techniques to handle skewed and long-tailed data

- Transformation, see Carroll and Ruppert (1988)
- Parametric regression
  - Generalized linear model (GLM), see Haberman and Renshaw (1996), Parametric survival model, see Lawless (2003) and GB2 regression, see Sun et al. (2008), Frees and Valdez (2008), Frees et al. (2008)
  - Random effects are use to account for heterogeneity and serial correlation

Quantile Regression

- First introduced by Koenker and Bassett Jr (1978)
- Advantages in long-tail regression modeling include easier interpretation, higher efficiency and robustness to outliers
- Longitudinal Quantile Regression
  - Jung (1996): quasi-likelihood method
  - Lipsitz et al. (1997): weighted generalized estimating equations
  - Koenker (2004): regularization method
  - Geraci and Bottai (2007): asymmetric Laplace density
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The regression quantiles $\beta(\tau)$ in the $\tau$th conditional quantile function $Q_{\tau}(y|x) = x'\beta(\tau)$ can be estimated by solving

$$\min_{\beta \in \mathbb{R}^k} \sum_{i=1}^{n} \rho_{\tau}(y_i - x'_i\beta).$$

Also, $\rho_{\tau}(u) = u(\tau - I(u \leq 0))$ is check function and $I(\cdot)$ is the indicator.

Asymmetric Laplace Distribution

$$f(y; \mu, \sigma, \tau) = \frac{\tau(1-\tau)}{\sigma} \exp\left( -\frac{y - \mu}{\sigma} \left[ \tau - I(y \leq \mu) \right] \right)$$

- Defined on $(-\infty, +\infty)$
- Location $\mu$, scale $\sigma$, skewness $\tau$ (Yu and Zhang (2005))
- Under $\mu = x'\beta$, the MLE with ALD($\mu, \sigma, \tau$) assumption results in regression quantiles (Yu et al. (2003))
- $E(y|x) = \mu(x) + \frac{\sigma(1-2\tau)}{\tau(1-\tau)}$
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Longitudinal Quantile Regression Model

- Use ALD\((\mu, \sigma, \tau)\) for marginals
- Use copula function to model intertemporal dependence
  \[
  f_i(y_{i1}, \ldots, y_{iT_i}) = c(F_{i1}, \ldots, F_{iT_i}; \phi) \prod_{t=1}^{T_i} f_{it}
  \]
- Parameterize \(\mu_{it} = x_{it}'\beta\) in ALD\((\mu, \sigma, \tau)\), then the log-likelihood function for \(i\)th insurer is shown as
  \[
  l_i = \ln \frac{\tau(1 - \tau)}{\sigma} - \frac{1}{\sigma} \sum_{t=1}^{T_i} \rho_{\tau}(y_{it} - x_{it}'\beta) + \ln c(F_{i1}, \ldots, F_{iT_i}; \phi)
  \]
- Quantile regression are preserved for marginals and we are interested in the \(\tau\) of best fit
Model Extension

Approach I: Rescaling

\[ Y_{it} = \frac{\text{Total Expenses}_{it}}{\text{Total Assets}_{it}}. \]

- allows one to compare different sized firm
- requires prediction of total assets

Approach II: Transformation

- Idea: transform data to ALD
- Normality-improving and variance-stabilizing (Pierce and Shafer (1986))
- To create new distributions (Bali (2003), Bali and Theodossiou (2008))
- We consider modulus transformation (John and Draper (1980)), IHS (Burbidge and Magee (1988)), modified modulus transformation (Yeo and Johnson (2000))
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<table>
<thead>
<tr>
<th>Covariate</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPW_P</td>
<td>Gross premium written of personal lines</td>
</tr>
<tr>
<td>GPW_C</td>
<td>Gross premium written of commercial lines</td>
</tr>
<tr>
<td>IRATIO</td>
<td>Cash and invested assets (net admitted)</td>
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<tr>
<td>LOSS_L</td>
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<tr>
<td>LOSS_S</td>
<td>Losses incurred for short tail lines of business</td>
</tr>
<tr>
<td>ASSET_CURR</td>
<td>Net admitted assets in current year</td>
</tr>
<tr>
<td>STOCK</td>
<td>Indicates if the company is a stock company</td>
</tr>
<tr>
<td>MUTUAL</td>
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Regression quantiles for intercept and GPW_P
Out-of-sample validation is based on the predictive density:

\[ f(y_{i,T+1} | y_i, \ldots, y_T) = \frac{c(F_{i1}(y_{i1}), \ldots, F_{iT+1}(y_{i,T+1}))}{c(F_{i1}(y_{i1}), \ldots, F_{iT}(y_{i,T}))} f_{i,T+1}(y_{i,T+1}) \]

Calculate the percentile of \( y_{i2006} \) by \( p_i = F(y_{i2006}) \) for \( i = 1, \ldots, n_h \), where \( F(\cdot) \) is the cdf of the predictive distribution.

\( p_i \) should be uniform if the model is well specified.
A residual is the company expense, controlled for company characteristics. A small residual means an inexpensive company. We look into residuals to identify cost efficient companies.

- We have no external measures to validate our notions of an “inexpensive” company but can look to A. M. Best Ratings
  - Ratings indicate the financial strength of an insurer
  - Not the same as the expense situation for a company
  - Still, a less expensive insurer tends to be more profitable, and thus has a healthier financial status and higher rating
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The average residuals over 2001-2005 are employed in the analysis.

Define the residual percentile as the ratio of the rank of an residual to the number of insurers.

A financially strong company will have low expenses, meaning that the percentiles of the distribution of expenses are small.

Counts of Insurers with Secure Rating

<table>
<thead>
<tr>
<th>Residual Percentile</th>
<th>Superior</th>
<th>Excellent</th>
<th>Good</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Copula</td>
<td>RE</td>
<td>Copula</td>
</tr>
<tr>
<td>0-10</td>
<td>51</td>
<td>42</td>
<td>83</td>
</tr>
<tr>
<td>10-25</td>
<td>78</td>
<td>69</td>
<td>126</td>
</tr>
<tr>
<td>25-50</td>
<td>119</td>
<td>125</td>
<td>197</td>
</tr>
<tr>
<td>&gt;50</td>
<td>96</td>
<td>108</td>
<td>459</td>
</tr>
<tr>
<td>Totals</td>
<td>344</td>
<td>344</td>
<td>865</td>
</tr>
</tbody>
</table>

The copula model outperforms the random effects model in classifying more insurers into higher efficiency range (top 50th percentile) for all categories of secure rating.
Model features:

- Introduces a quantile regression model for longitudinal data
- Captures heavy tailed nature of insurance company expenses
- Allows for negative values of expenses
- Captures intertemporal dependence of expenses through a copula function
- Allows for covariates for expenses
- Provides a predictive distribution for insurer’s expenses

Future work:

- Will look at each type of expenses
- Will examine the efficiency of insurers using more formal “stochastic frontier” models
Transformation Method

- Generic form $Y^{(\lambda)} = \psi(Y, \lambda)$

- Three transformations
  - Modulus
    \[
    y^{(\lambda)} = \begin{cases} 
    \text{sign}(y) \left\{ (|y| + 1)^{\lambda} - 1 \right\} / \lambda, & \lambda \neq 0 \\
    \text{sign}(y) \log(|y| + 1), & \lambda = 0
    \end{cases}
    \]
  
  - IHS
    \[
    y^{(\lambda)} = \frac{\sinh^{-1}(\lambda y) / \lambda}{1 / \lambda} = \ln(\lambda y + (\lambda^2 y^2 + 1)^{1/2})^{1/\lambda}
    \]

  - Modified Modulus
    \[
    y^{(\lambda)} = \begin{cases} 
    (y + 1)^{\lambda} - 1 \big/ \lambda, & y \geq 0, \lambda \neq 0 \\
    \log(y + 1), & y \geq 0, \lambda = 0 \\
    -\{(-y + 1)^{2-\lambda} - 1\} / (2 - \lambda), & y < 0, \lambda \neq 2 \\
    -\log(-y + 1), & y < 0, \lambda = 2
    \end{cases}
    \]
Analysis of Rescaled Expenses

- Marginal distribution

- Intertemporal dependence (Define $\hat{e}_{it} = (y_{it} - \hat{\mu}_{it})/\hat{\sigma}$, $\hat{\mu}_{it} = x_{it}'\hat{\beta}$)

<table>
<thead>
<tr>
<th></th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
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<tbody>
<tr>
<td>2001</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td>0.857</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2003</td>
<td>0.774</td>
<td>0.852</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2004</td>
<td>0.686</td>
<td>0.754</td>
<td>0.823</td>
<td>1.000</td>
<td></td>
<td></td>
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<tr>
<td>2005</td>
<td>0.642</td>
<td>0.692</td>
<td>0.740</td>
<td>0.824</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td>0.625</td>
<td>0.667</td>
<td>0.691</td>
<td>0.759</td>
<td>0.844</td>
<td>1.000</td>
</tr>
</tbody>
</table>
## Analysis of Rescaled Expenses

### Estimates for the Longitudinal Quantile Regression Model with Different Copulas

<table>
<thead>
<tr>
<th></th>
<th>Gaussian Estimates</th>
<th>Gaussian t-stat</th>
<th>Student Estimates</th>
<th>Student t-stat</th>
<th>RE Estimates</th>
<th>RE t-stat</th>
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</thead>
<tbody>
<tr>
<td>SIGMA</td>
<td>0.0282</td>
<td>54.62</td>
<td>0.0281</td>
<td>222.50</td>
<td>0.1843</td>
<td>23.42</td>
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<tr>
<td>TAU</td>
<td>0.2130</td>
<td>54.87</td>
<td>0.2129</td>
<td>63.47</td>
<td>0.984</td>
<td>3.31</td>
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<tr>
<td>BINT</td>
<td>0.0983</td>
<td>245.22</td>
<td>0.0984</td>
<td>242.41</td>
<td>0.1843</td>
<td>23.42</td>
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<tr>
<td>BLOSS_L</td>
<td>0.0287</td>
<td>5.99</td>
<td>0.0228</td>
<td>5.63</td>
<td>0.0389</td>
<td>3.51</td>
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<tr>
<td>BLOSS_S</td>
<td>0.0255</td>
<td>11.23</td>
<td>0.0169</td>
<td>7.29</td>
<td>0.0201</td>
<td>3.31</td>
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<tr>
<td>BPREM_P</td>
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<td>5.98</td>
<td>0.0168</td>
<td>5.57</td>
<td>0.0158</td>
<td>2.74</td>
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<td>BPREM_C</td>
<td>0.0092</td>
<td>8.11</td>
<td>0.0095</td>
<td>7.15</td>
<td>0.0061</td>
<td>1.86</td>
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<td>BASSET_CURR</td>
<td>-0.0057</td>
<td>-7.77</td>
<td>-0.0043</td>
<td>-6.22</td>
<td>-0.0108</td>
<td>-7.31</td>
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<tr>
<td>BIRATIO</td>
<td>-0.0639</td>
<td>-83.12</td>
<td>-0.0638</td>
<td>-162.86</td>
<td>-0.0423</td>
<td>-6.05</td>
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<tr>
<td>BGROUP</td>
<td>-0.0226</td>
<td>-7.54</td>
<td>-0.0225</td>
<td>-60.65</td>
<td>-0.0369</td>
<td>-10.21</td>
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<td>BSTOCK</td>
<td>0.0200</td>
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<td>0.0201</td>
<td>20.19</td>
<td>0.0276</td>
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<td>BMUTUAL</td>
<td>0.0706</td>
<td>171.84</td>
<td>0.0705</td>
<td>24.39</td>
<td>0.0487</td>
<td>6.95</td>
</tr>
<tr>
<td>RHO1</td>
<td>0.8371</td>
<td>205.71</td>
<td>0.8390</td>
<td>134.13</td>
<td>0.8371</td>
<td>205.71</td>
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<tr>
<td>RHO2</td>
<td>0.7405</td>
<td>106.76</td>
<td>0.7561</td>
<td>152.60</td>
<td>0.6685</td>
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<tr>
<td>RHO3</td>
<td>0.6643</td>
<td>66.42</td>
<td>0.6893</td>
<td>87.63</td>
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<td>4.57</td>
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<tr>
<td>RHO4</td>
<td>0.5979</td>
<td>42.09</td>
<td>0.6416</td>
<td>47.14</td>
<td>0.4953</td>
<td>6.95</td>
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<tr>
<td>TDF</td>
<td>6.3453</td>
<td>31.34</td>
<td></td>
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</tbody>
</table>

- LogLikelihood: 14,090.68, 15,741.96, 13,150.55
- AIC: -28,149.37, -31,449.92, -26,269.10

### Histogram and QQ plot of residuals of random effect model
Analysis of Transformed Expenses

- QQ plots of transformed asymmetric Laplace distributions
  - Modulus
  - IHS
  - Modified Modulus

- QQ plots of transformed normal distributions
  - Modulus
  - IHS
  - Modified Modulus
### Estimates for $t$-Copula Models with Different Dependence Structure

<table>
<thead>
<tr>
<th></th>
<th>AR1</th>
<th>Exchangable</th>
<th>Toeplitz</th>
<th>Unstructured</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>$t$-stat</td>
<td>Estimate</td>
<td>$t$-stat</td>
</tr>
<tr>
<td>SIGMA</td>
<td>0.0028</td>
<td>78.03</td>
<td>0.0019</td>
<td>123.08</td>
</tr>
<tr>
<td>TAU</td>
<td>0.6777</td>
<td>117.20</td>
<td>0.8268</td>
<td>400.44</td>
</tr>
<tr>
<td>LAMBDA</td>
<td>245.4257</td>
<td>704.66</td>
<td>244.0306</td>
<td>675.03</td>
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